Team Control Number

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Problem Chosen

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ShuWei Cup

Summary

This paper focuses on the aging and medical and health needs, combined with the competition and cooperation between public hospitals and private hospitals. To explore the process and queuing methods of patients' hospital treatment, and put forward suggestions for the reform and improvement of relevant medical management departments.

Problem 1: This section uses 1994-2010 data to build ARIMA model to calculate the population mortality rate, and then uses the population age migration model to calculate the number of elderly people of all ages. The medical and medical data mainly come from 2010-2017 China Health Statistics Yearbook, and uses matlab2018 programming to build a grey prediction model GM (1, 1) to predict the medical resources of China Source and service requirements.

Problem 2: Seafood and beer are one of the important causes of cardiovascular and cerebrovascular diseases. In this section, the data of prehospital emergency cardiovascular and cerebrovascular diseases in Shandong public hospital from 2011 to 2018 are used, and the single factor grey GM (1,1) model is used to successfully predict the total number of prehospital emergency cardiovascular and cerebrovascular diseases patients in 2019-2023.

Problem 3: The internal medicine patients are relatively dense, and the mobility is also relatively large. Therefore, taking the outpatient department of internal medicine as an example, the queuing theory is used to build a queuing model for the outpatient department structure of the hospital. This paper discusses the M / M / 1 Queuing System with feedback, gives the system state transition diagram and transition rate matrix, and gives the steady-state results of the system from the transition rate matrix, including the distribution of the system team leader, the number of patients waiting in line, the average team leader, the average waiting team leader, etc., which provides some valuable reference data for the outpatient staffing and the distribution of patients.

Problem4: This paper uses the maximization method to study the cooperative decision-making problem of resource sharing and utilization, establishes a decision-making model of medical service product output, and compares the optimal decision-making output of medical service market and medical service product provided by private hospitals and public hospitals when they make joint decision-making and independent decision-making respectively.

Problem5:The gray clustering model is used to classify the hospitals which close to each other, build and share then resources and information.

Key word: ARIMA model;grey model;M / M / 1 Queuing System;decision-making model

Content

1. Introduction

1.1 Background

The medical industry is one of the most important pillar industries of the whole country and the whole society. In recent years, in order to adapt to the rapid development of China's economy and the aging of the country, the medical and health fields have also been gradually transformed. The fast-growing economy has improved the people's quality of life, but it also inevitably brings about the rejuvenation of some diseases and medical needs. Rapid growth and diversified development have also led to an increase in the number of private hospitals. Higher requirements have been placed on the reform and improvement of hospitals. In general, it is necessary to ensure the effective use of resources, the allocation of medical personnel is more reasonable, and the prevention of diseases should also be emphasized. This means the need to use mathematical models to solve practical medical problems.

1.2 Work

Task 1: According to the statistics of residents' income, population age structure and economic development level in the relevant statistical analysis data of the National Bureau of Statistics, reasonably predict China's aging trend and the medical needs of residents.

Task 2: Take a province as an example to analyze the most common diseases in the province in the future and provide recommendations for the overall development of the main public hospitals in the province.

Task 3: Different types of patients may need to be examined differently in the hospital; different inspection items may be distributed in different locations, and the number of people in the queue may vary greatly. Please propose a general queuing theory and its related optimal queuing method for this queuing problem.

Task 4: Combine the complex cooperation and competition between private hospitals and public hospitals, please propose the best cooperation and competition strategy between multiple hospitals.

Task 5: Write a 1-2 page proposal for the relevant medical management department and provide a reference for the development of the “fourteenth five-year plan”.

2. Problem analysis

2.1 Analysis of question one

Based on the data of age and gender mortality in China from 1994 to 2010, the ARIMA prediction model was constructed to predict the age and gender mortality rate in China from 2011 to 2050. According to the formula, the predicted age and gender survival rate per 5 years from 2015 to 2050 were obtained. Based on this, the sixth national census data of 2010 will be used to construct the population age migration model, and the number of elderly people by age and sex in China will be predicted from 2015 to 2050. Use MATlAB2018 to program the gray prediction model GM(1,1) to predict the demand for medical resources and services in China.

2.2 Analysis of question two

Based on the data of pre-hospital emergency cardiovascular and cerebrovascular diseases in public hospitals in Shandong Province from 2011 to 2018, this study will analyze and predict the current situation of cardiovascular and cerebrovascular diseases in Shandong Province and the situation in the next five years, using single factor gray GM (1, 1) Model, forecasting is done in Excel 2010,The model comparison uses mean absolute error (MAE), average absolute percentage error (MAPE), and root mean square error (RMSE).

2.3 Analysis of question three

Question 3 requires that different types of patients may need to be examined differently in the hospital; different inspection items may be distributed in different locations, and the number of people in the queue may have a large gap, etc., and a general queuing theory is proposed. And the best method; for this problem, this article takes the internal medicine clinic as an example to use the queuing theory to establish a queuing model for the hospital outpatient structure. Considering the patient's condition, the M/M/1 queuing system with feedback is discussed. The system state transition diagram and the transfer rate matrix are given, and the steady-state results of the system are given by the transfer rate matrix, including the system leader and queue. Waiting for the distribution of the number of patients, the average captain, the average waiting for the captain, etc.

2.4 Analysis of question four

Question 4 requires the cooperation and competition between private hospitals and public hospitals, and proposes the best cooperation and competition strategy among many hospitals. In response to this problem, this paper uses the maximization method to deeply study the cooperation and competition between private hospitals and public hospitals. Strategy, through research on the cooperative decision-making problem of resource sharing and utilization between private hospitals and public hospitals, establish a decision model for the output of medical service products, compare the private hospitals and public hospitals to make joint decisions and the independent decision-making, the medical service market is optimal and they provide Optimal decision making for medical service products.

2.5Analysis of question five

Using the grey clustering model to classify the local public and private hospitals, and maintain the development environment of different ownership medical institutions in terms of laws and policies, build information collaboration platform, resource sharing and medical information sharing.

3.Symbol and Assumptions

3.1 Symbol Description

Symbol	Significance
μ	Endogenous control gray number
α	Development gray number
C	Posterior variance
P	Minimum error probability
MAE	Average absolute error
RMSE	Root mean square error
MAPE	Average absolute percentage error
π_k	Stationary probability
L_q	Number of patients waiting in line
W	Waiting time in queue
k	Number of patients
π_1	Private hospital's benefit function
π_2	Public hospital benefit function
Q_1	Number of medical services provided by private hospitals
Q_2	Number of medical services provided by public hospitals
ΔQ_1	Improvement of medical service supply in private hospitals
ΔQ_2	Improvement of medical service supply in public hospitals

3.2 Fundamental assumptions

1.Assume that the elderly in China move into and out of balance

2.Regardless of major accidents

3.Residual sequence is irrelevant

4.The medical services provided by private hospitals and public hospitals are both price elastic, and their prices and demands meet the linear relationship

4.Model and solution

4.1Model establishment and solution of problem one

4.1.1Data Sources

This data is derived from the national 1% population sample survey in 1995 and 20055, the 5th national census in 2000, and the 6th national census data in 2010. Since the data before 1994 is age-independent, This paper selects the data from 1994 to 2010 and uses the 6th census data in 2010 as the prediction base. The medical and medical resources data mainly comes from the 2010-2017 National Traditional Chinese Medicine Statistics Extract. The service data is mainly from the 2018 China Health and Health Statistical Yearbook. The data from 2010-2018 is used for GM (1,1) modeling analysis. The data for 2019-2020 is forecast data.

4.1.2Age and sex mortality prediction

This study will use the ARIMA model in time series analysis technology to use the 1994-2010 China population mortality data to fit and predict the mortality rate of China's age and sex population. The death rate of Chinese males aged 75-79 in 1994-2010 For example, during this period, the mortality rate of males aged 75-79 in China showed a downward trend, which was an unsteady sequence.

After the first-order difference of the sequence, the trend effect is eliminated and the stationary time series is obtained. The software preliminary judgment model is ARIMA (2, 1, 0). Based on the Q statistic of the fitted residuals and the sequence-related B-J test results, as well as the AIC criteria, the surface selected model is appropriate.

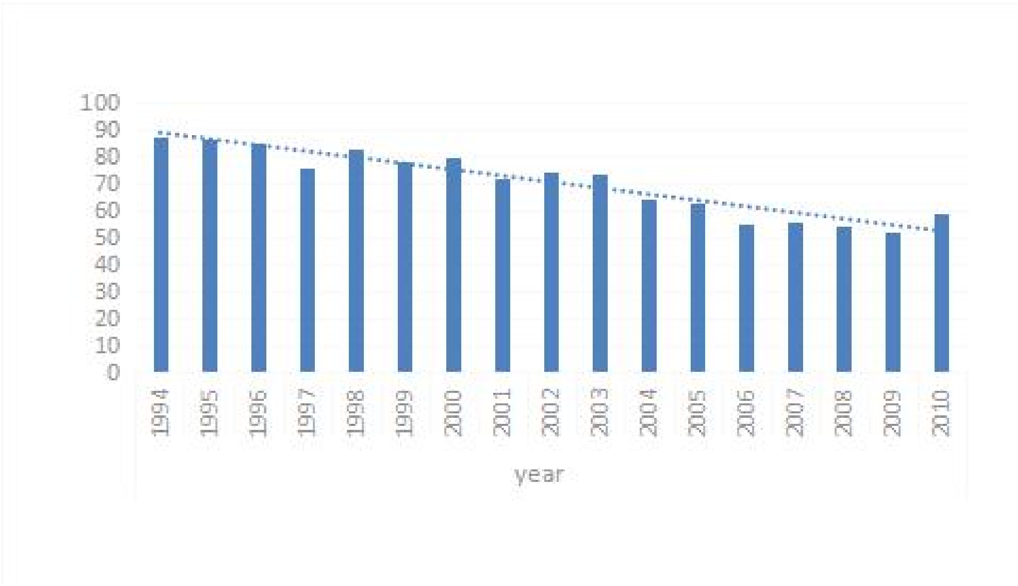


Fig 1 Mortality rate of men aged 75-79 in 1994-2010

Further predicting the mortality rate in 2011-2016, the prediction results are basically consistent with the actual results, see Table 1

Table 1Comparison of ARIMA (2,1,0) model and actual results

TIME	Actual value(x^2)	Predictive value(x^2)	95% CI	Difference (d)	d/x^2
2011	59.55	57.34	48.48~66.30	2.21	0.04
2012	60.26	59.86	48.28~71.49	0.4	0.01
2013	61.64	58.84	43.08~74.59	2.8	0.05
2014	58.24	59.85	41.41~78.28	1.61	0.03
2015	53.60	59.33	37.97~80.70	5.73	0.10
2016	55.15	59.75	36.06~83.45	4.60	0.08

According to the AIC criterion, this model does not necessarily make the AIC of each age group or female each age minimum. According to the analysis results of the predicted values, the overall prediction effect is better. ARIMA (2, 1, 0) model.

4.1.3Survival rate calculation

If the prediction interval is 5 years, 5 mortality prediction values of age group (i, i +5) can be obtained for each 5-year-old age group ,denoted by $\mu_{i1},......,\mu_{i5}$,

$\exp(\sum_{x=1}^{5}\mu_{i5})$ As a prediction of 5-year survival rate, denoted byS（t,i）.

4.1.4Elderly population prediction

An age-calculation model for predicting the number of elderly populations. This model can transfer the population of a certain age group in a certain year to the next age group in the next year under the corresponding probability conditions. The main feature is based on population changes. The endogenous mechanism, the principle of migration is rigorous, the method is simple and easy, and it has a fairly high accuracy and is widely used in population prediction. In order to improve data and forecast accuracy, the last census data for 2010 is used as the basic data. By 2060, 60-year-olds have been born, regardless of birth factors, (t, t+5) years (i+5) The old age population is:N(t+5,i+5)=N(t,i)S(t,i)....

The forecast results show that the total number of elderly people aged 60 and over in China from 2015 to 2050 is 232 million, 275 million, 333 million, 398 million, 446 million, 463 million, 473 million and 489 million.

Table 2 Prediction results of the elderly population in China during 2015-2050

years	group	60~64	65~69	70~74	75~79	80~84	85~89	》90
2015	man	3952	2804	1870	1371	840	361	134
	female	3938	2779	1910	1470	1031	524	254
2020	man	3780	3713	2528	1560	1017	510	230
	female	3706	3796	2606	1694	1205	725	408
2025	man	4949	3552	3347	2108	1157	618	340
	female	4962	3572	3561	2312	1389	846	588
2030	man	5785	4651	3202	2791	1565	703	435
	female	5923	4782	3320	3158	1895	976	733
2035	man	5446	5436	4192	2670	2072	950	513
	female	5470	5613	4486	2971	2589	1332	868
2040	man	4435	5117	4900	3496	1982	1258	665
	female	4506	5227	5265	3978	2436	1819	1129

2045	man	4532	4167	4613	4086	2595	1203	876
	female	4737	4343	4945	4668	3262	1712	1520
2050	man	5683	4258	3757	3874	3033	1575	926
	female	5977	4565	4074	4386	3828	2292	1618

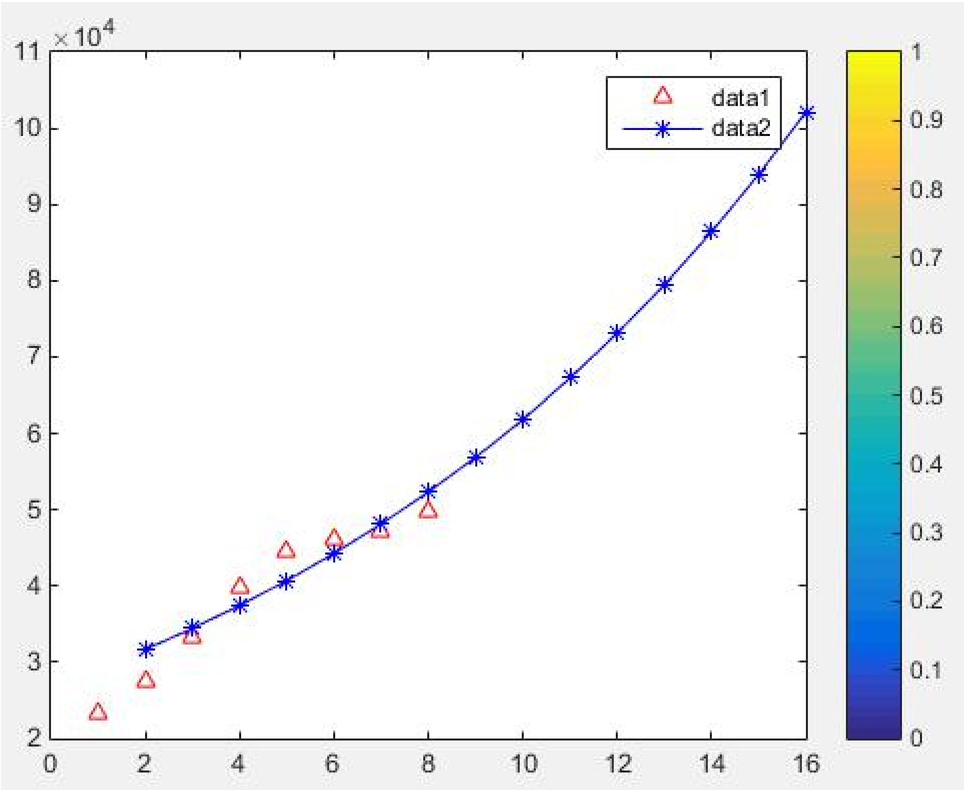


Fig 2 Fitting curve

4.1.5Model introduction

Grey Model,By using the accumulation and subtraction methods to generate and process the original data, find the data change law to generate a regular data sequence and then establish the corresponding differential equation model to achieve trend prediction of the data. The model type includes GM. (1,1), GM(1,n), etc. The GM(1,1) model based on MATLAB 2018 has the characteristics of small sample size and high short-term prediction accuracy compared with the ARIMA prediction model established by Eviews software. Data resource prediction with insufficient prediction integrity has great practicability. This chapter uses GM (1,1) model for predictive analysis. Its model function is

$$X^{(1)}(\mathrm{k}+1)=(X_{(1)}^{(0)}-\mu/\alpha)e^{-ak}+\mu/\alpha \tag{1}$$

Table 3 Predictive model fitting accuracy

Precision level	Posterior variance	error probability	α
Good	C≤0.35	0.95≤P	$-\alpha \leq 0.3$

qualified	0.35<C≤0.50	0.80≤P<0.95	0.3<-α ≤0. 50
reluctantly	0.50<C≤0.65	0.70≤P<0.80	0.50<-α ≤0. 80
Failed	0.65<C	P<0.70	0. 80<-α ≤1

The data was entered and sorted using Microsoft Excel 2010, and the GM (1,1) model was used to study and predict the data through MATLAB 2018 software.

4.1.6Model building process

1. Generate a first-order accumulation sequence:

To reduce the volatility and randomness of the sequence,Raw sequence

$x^{(0)}(k) = (x^{(0)}(1),\ x^{(0)}(2),\ \cdots,\ x^{(0)}(n))$, $X^{(1)}$ is 1－AGO sequence of $X^{(0)}$, $X^{(1)} =$

$(x^{(1)}(1),\ x^{(1)}(2),\ \cdots,\ x^{(1)}(n))$, $x^{(1)}(k)=\sum_{i=1}^{k} x^{(0)}(i),(i=1,2,\cdots,n)$ (2)

2. Generate a mean sequence

$Z^{(1)}$is the sequence of the nearest mean of X(1).

$$z^{(1)}(k)=\frac{1}{2}(x^{(1)}(k)+x^{(1)}(k-1)),(k=1,2,\cdots,n) \tag{3}$$

3. Calculation

Establish the first-order unary differential equation GM(1,1) , the basic formula is:

$$\alpha=\left\{(N-1)\left[\sum_{k=2}^{N} z_{(k)}^{2}\right]+\left[\sum_{k=2}^{N} z_{(K)}^{2}\right]\left[\sum_{K=2}^{N} X_{(K)}^{(0)}\right]\right/ D \tag{4}$$

$$\mu=\left\{\left[\sum_{k=2}^{N} z_{(k)}^{2}\right]\left[-\sum_{k=2}^{N} X_{(k)}^{(0)}\times Z_{k}\right]+\left[\sum_{k=2}^{N} Z_{(k)}^{2}\right]\left[\sum_{k=2}^{N} X_{(k)}^{(0)}\right]\right/ D \tag{5}$$

$$D=(N-1)\left[\sum_{k=2}^{N} Z_{(K)}^{2}\right]+\left[\sum_{K=2}^{N} Z_{(K)}^{2}\right]^{2} \tag{6}$$

Calculate the original sequence standard deviation S1 and the residual standard deviation S2, and then obtain the posterior variance C value and the minimum error probability P value. Based on the C value and the P value, the residual model test and the fitting effect analysis are performed. The smaller the value and the larger the P value, the better the fit.

$$S_{1}=\sqrt{\sum_{k-1}^{n}\left[X^{0}(k)-\overline{X}\right]\frac{1}{n}} \tag{7}$$

$$S_2 = \sqrt{\sum_{k-2}^{n}\left[E(k) - \overline{E}\right]\frac{1}{n\text{-}1}} \tag{8}$$

$$C = \frac{S_1}{S_2} \tag{9}$$

$$P = P\ (\left|E\ (k) - \overline{E}\ \right|) < 0.6745 S_1 \tag{10}$$

4.1.7Forecast result

Based on MATLAB 2018, the four prediction unit data of hospital institutions, beds, discharges, and total number of patients were modeled, and the μ and α values of each prediction unit were obtained and brought into the GM (1,1) model. Order one-dimensional differential equation:

$$X^{(1)}(\mathrm{k}+1) = (X_{(1)}^{(0)} - \mu/\alpha)e^{-ak} + \mu/\alpha \tag{11}$$

Table 4 Predictive model result

Forecasting unit	GM(1,1) first-order linear differential equation
Institutions	$X^{(1)}(\mathrm{k}+1) = 56{,}785.80e^{0.0552k}$ -53,553.80
Bed	$X^{(1)}(\mathrm{k}+1) = 5{,}874{,}114.64e^{0.0911k}$ -5,396,562.64
Number of discharged	$X^{(1)}(\mathrm{k}+1) = 16{,}050.94e^{0.0953k}$ -14,775.24
Total patients	$X^{(1)}(\mathrm{k}+1) = 1{,}059{,}494.21e^{0.0641k}$ -998,230.11

Table 5 Predictive result for 2010－2020

years	Hospital	Bed	discharged	Total patients
2010	3232	477552	1275	61264
2011	3223	559592	1605	70139
2012	3406	612880	1765	74783
2013	3599	671243	1942	79735
2014	3803	735164	2136	85015
2015	4019	805171	2350	90644
2016	4247	881846	2585	96647
2017	4488	965822	2843	103046
2018	4843	1057795	3128	109870

2019	5013	1158526	3441	117145
2020	5296	1268850	3785	124902

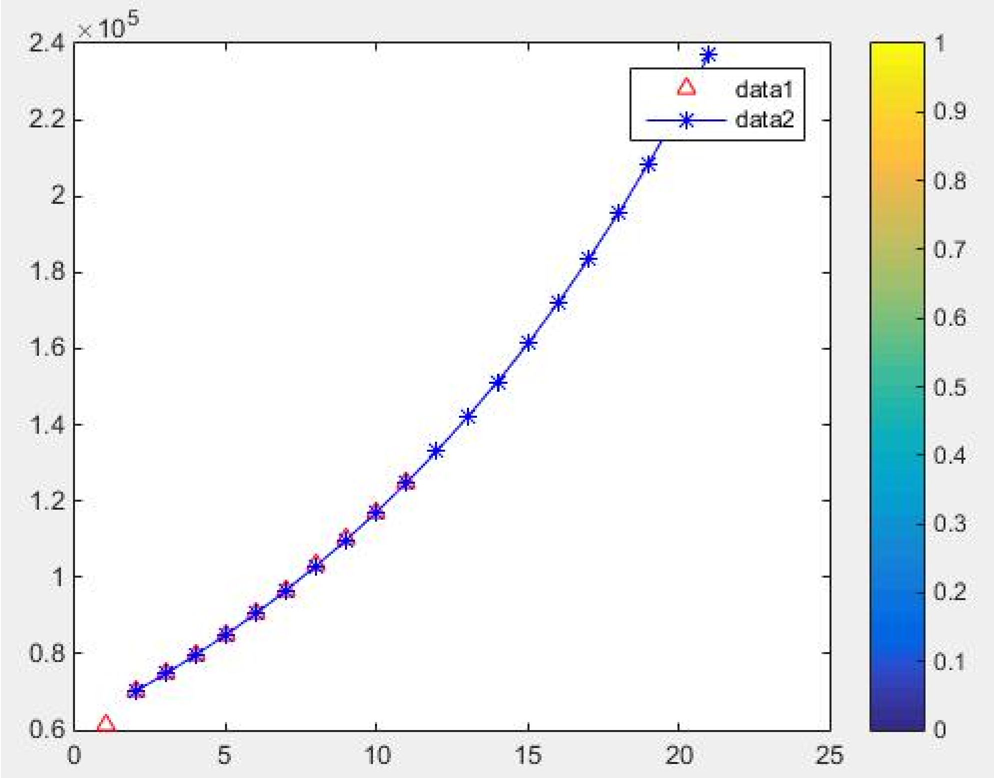


Fig 3 Fitting curve

4.1.8Conclusion

1. The number of elderly people is huge, showing an increasing trend year by year and the growth rate is accelerating.
2. The sex ratio of the elderly population is less than 100. The older the age group, the lower the sex ratio.
3. The elderly population is developing in the direction of aging, and the growth rate of the elderly population is accelerating.
4. People's demand for medical resources and services is steadily rising.

4.2Model establishment and solution of problem two

4.2.1Data Sources

According to the study, among the patients with cardiovascular and cerebrovascular diseases in pre-hospital emergency in Shandong Province from 2011 to 2018, there were 49,120 males and 35,760 females, with a male-female ratio of 1.37:1. The male incidence rate (435.11/100,000) was significantly higher than that of females (334.62/100,000), and the difference was statistically significant ($\chi 2$=1 209.098, P<0.001), and the male morbidity rate was 1.30 times that of female sex. The incidence of cardiovascular and cerebrovascular diseases in both sexes showed an increasing trend. The data showed ($\chi 2$=74.540, P<0.001; $\chi 2$=27.054, P<0.001). See

Fig 4 for details .

Fig 4 patients in Shandong Province from 2011 to 2018

According to statistics, the average age of onset of cardiovascular and cerebrovascular diseases in the province is (63.83±16.77) years old. From the ratio of composition of all ages, the proportion of people aged ≥60 years old is the largest (62.50%), followed by the population of 35-59 years old (32.55%), and the proportion of people aged 18-34 is relatively small (4.15%). The proportion of people ≤17 years old is the least (0.79%). Correspondingly, the incidence rate of each age group is ≥60 years old (2195.60/100,000), 35~59 years old (355.89/100,000), 18~34 years old (69.44/100,000), ≤17 years old (24.85/100,000) There was a significant difference ($\chi2$=183 659.616, P<0.001). The incidence of ≥60 years old population was significantly higher than other age groups, and the ≤17 year old population had the lowest incidence rate.

4.2.2Model building process

Building a predictive model：

$$x(t) = 122987.70e^{0.07t} - 112708.70 \tag{1}$$

The obtained model is tested by post-test to obtain S1=518.16, S2=1 503.12, C=S1/S2=0.34, P=1.00, and the model accuracy reaches one level (good), indicating that the model has high precision and can be used for extrapolation prediction. . Let X1 be the total population, X2 be the per capita GDP, X3 be the per capita disposable income, X4 for per capita medical care, X5 for the elderly population, and X6 for the number of medical staff. The gray GM(1,1) model is established for X1 and X3 respectively, and the average relative error is 0.33% and 0.92%, respectively. The accuracy of X3 is one level (good), which can be used for prediction, and the accuracy of X1 is four levels (Unqualified), cannot be predicted

The buffer operator is used for processing,

$$x(k)d = \frac{1}{n-k+1}\left[x(k) + x(k+l) + ...x(n)\right] \tag{2}$$

and the gray GM(1,1) model is established for the second-order buffer sequence. The average relative error is 0.03%, and the precision reaches one level (good), which can be used for prediction. The predicted values of X1 and X3 in 2018-2023 are obtained by using the established one-factor gray GM(1,1) model. By substituting the predicted values of X1 and X3 into the regression equation, the total number of patients with cardiovascular and cerebrovascular diseases before hospitalization in 2019-2023 can be predicted.

4.2.3Comparison of prediction results and accuracy

Table 6 Compare results

Years	Actual value	Predictive value	Residual	Relative error
2011	102790	102786	4	-0.0001
2012	92660	86129	6531	-0.079
2013	92280	92210	70	-0.0028
2014	95550	98630	-3080	0.0305
2015	105400	105340	60	-0.0029
2016	103420	112221	-8801	0.0898
2017	120140	120600	-460	0.0037
2018	136000	129030	6790	-0.513
2019	-	138040	-	-
2020	-	147710	-	-
2021	-	158030	-	-
2022	-	169091	-	-
2023	-	180990	-	-

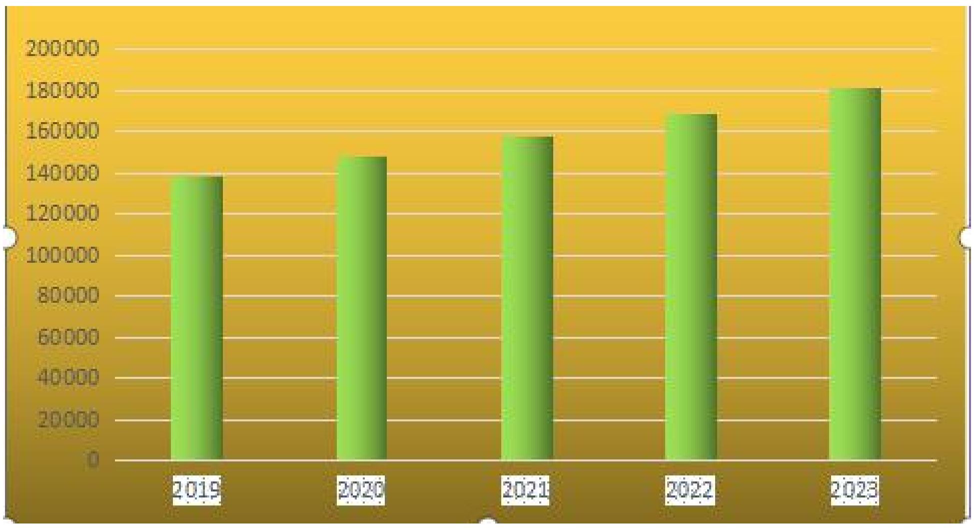


Fig 5 Compare results

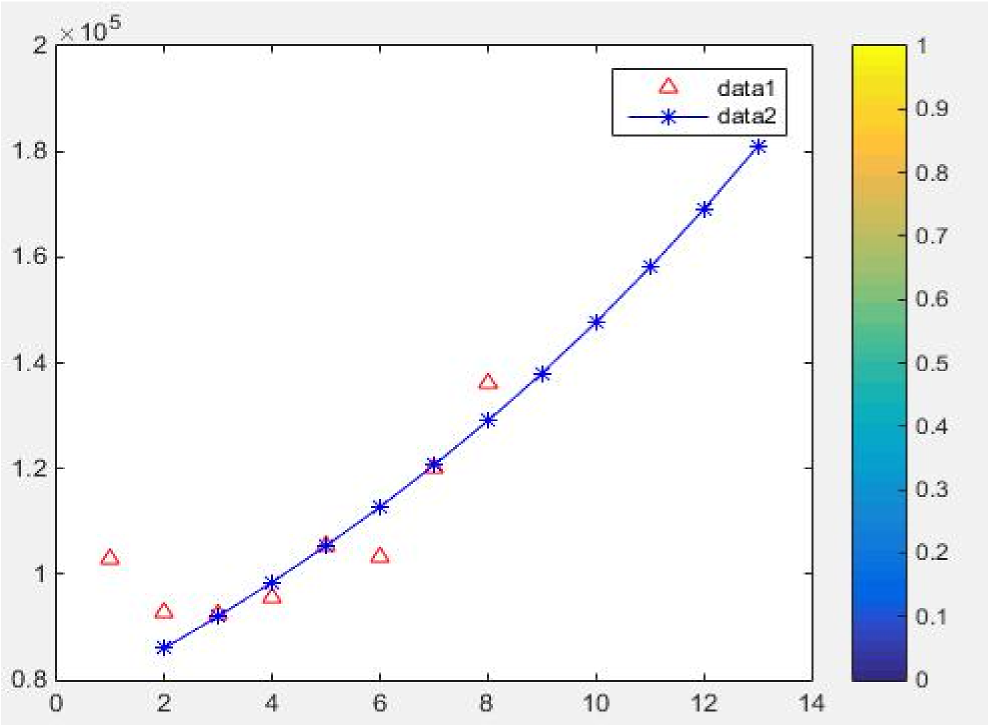


Fig 6 Compare curve

Cardiovascular and cerebrovascular diseases in China have the characteristics of high prevalence, high self-disability, high mortality, high recurrence rate, and the incidence tends to be younger. According to statistics, 2 out of every 5 deaths are due to cardiovascular and cerebrovascular diseases. Diseases, so for public hospitals, we must improve the first-aid mechanism and diagnosis process of cardiovascular and cerebrovascular diseases, increase the number of beds, and publicize the daily care of cardiovascular and cerebrovascular diseases. In addition, the drugs commonly used for cardiovascular and cerebrovascular diseases should be appropriately lowered in price. Respond to growing patients.

4.3Model establishment and solution of problem three

The topic requires that different types of patients may need to be examined differently in the hospital; different inspection items may be distributed in different locations, and the number of people in the queue may have a large gap, etc., and a general queuing theory and The best way. This article takes the internal medicine clinic as an example to adopt the queuing theory, and establishes a queuing model for the hospital outpatient structure. Considering the patient's condition, the M/M/1 queuing system with feedback is discussed. The system state transition diagram and the transfer rate matrix are given, and the steady-state results of the system are given by the transfer rate matrix, including the system leader and queue. Waiting for the distribution of the number of patients, the average captain, the average waiting for the captain, etc.

4.3.1Model establishment

The model is as follows:

(1) The arrival of the patient is random, assuming that the patient arrives in a possion flow with a mean of λ.

(2) Patients receive medical treatment in the order of first-come, first-served service during medical treatment, and the doctor leaves on their own after diagnosis.

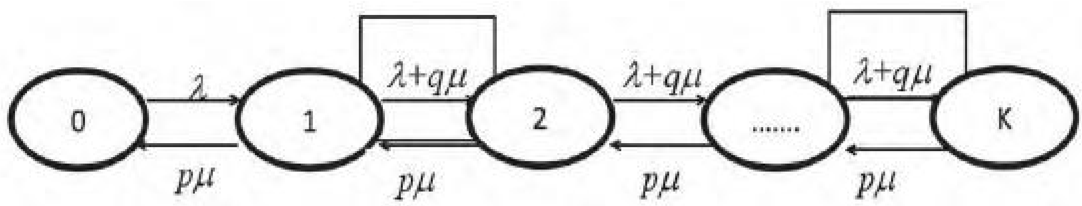
(3) The service time of the patient's diagnosis is subject to a negative exponential distribution with a mean of μ.

(4) Because different patients have different severity of illness, some patients need further examination, and then return to the clinic for further diagnosis. For example, some patients need further ultrasound to check for B-ultrasound or color ultrasound, etc., and they need to continue to queue after returning. Therefore, it is assumed that after the patient has completed the diagnosis, he or she leaves the clinic with probability q, does not return, and feeds back to the team with a probability p for further diagnosis, where q+p=1.

The arrival time interval and service time of all patients are independent of each other. Taking the normal working hours of the hospital as a reference, it is assumed that the number of patients arriving at normal working hours is sufficient, that is, the patient source is assumed to be infinite. The length of the team leader in the queuing system at t time (ie the number of patients waiting for medical treatment) is represented by L(t), {L(t), t≥0} is a Markov process.

The state space of the model is:{Ω=（0）,（1）,（2）,…,（K）,…,}

The state transition diagram for this system is as follows:



The transfer rate matrix Q of the system:

$$Q=\begin{bmatrix} -\lambda & \lambda & 0 & 0 & 0 & \cdots & 0 \\ p\mu & -(\lambda+\mu) & \lambda+q\mu & 0 & 0 & \cdots & 0 \\ 0 & p\mu & -(\lambda+\mu) & \lambda+q\mu & 0 & \cdots & 0 \\ 0 & 0 & p\mu & -(\lambda+\mu) & \lambda+q\mu & \ddots & 0 \\ 0 & 0 & 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \vdots & \vdots & \cdots & \ddots & -(\lambda+\mu) & \lambda+q\mu \\ 0 & 0 & 0 & \cdots & 0 & \ddots & \ddots \end{bmatrix} \quad (1)$$

4.3.2Steady state result of the system

The changes in the number of indicators in the outpatient system of the hospital are basically the same. We believe that the outpatient system is already in a state of statistical balance, that is, a stationary state. When the system reaches steady state, remember:

$$\pi_k = \lim_{t\to\infty} P\{L(t)=k,\}, k \ge 0, \tag{2}$$

The probability of writing the segment vector is:

$$\prod = (\pi_0,\ \pi_1, \cdots, \pi_k, \cdots) \tag{3}$$

Under steady-state conditions, assuming that the number of patients arriving is sufficient, the steady-state distribution of the QBD process {L(t), t≥0} can be expressed as:

$$\begin{cases} \pi_0 = 1 - \dfrac{\lambda}{(p-q)\mu} \\ \pi_k = \pi_0 (\dfrac{\lambda + q\mu}{p\mu})^{k-1} \dfrac{\lambda}{p\mu}, k = 0,1,\cdots,K,\cdots \end{cases} \tag{4}$$

Known by steady conditions:

$$\pi Q = 0,\ \ \pi e = 1 \tag{5}$$

That π satisfies the following equations:

$$-\lambda\pi_0 + p\mu\pi_1 = 0 \tag{6}$$

$$\lambda\pi_0 - (\lambda+\mu)\ \pi_1 + p\mu\pi_2 = 0 \tag{7}$$

$$(\lambda + q\mu)\ \pi_{i-1} - (\lambda+\mu)\pi_i + p\mu\pi_{i+1} = 0,\ \ i = 2,3,\cdots k-1,\cdots \tag{8}$$

$$\pi_0 e_0 + \sum_{i=1}^{K} \pi_i e_1 = 1 \tag{9}$$

Available from equation (6)(7)(8):

$$\pi_1 = \frac{\lambda}{p\mu}\pi_0\ \pi_2 = \frac{\lambda+q\mu}{p\mu}\pi_1 = \frac{\lambda+q\mu}{p\mu}\cdot\frac{\lambda}{p\mu}\pi_0\ \pi_{i+1} = \frac{1}{p\mu}\left[(\lambda+\mu)\pi_i - (\lambda+q\mu)\pi_{i-1}\right]$$

$$\pi_k = \pi_0\left(\frac{\lambda+q\mu}{p\mu}\right)^{k-1}\frac{\lambda}{p\mu}, k = 1,\cdots,K,\cdots \tag{*}$$

Substituting the above (*) into equation (9):

When $\dfrac{\lambda+q\mu}{p\mu} < 1$ The system is in steady state, $\pi_0 = 1 - \dfrac{\lambda}{(p-q)\ \mu}$.

The distribution of the number of patients waiting in the system for queuing in the steady state is Lq:

$$P\left\{L_q = k\right\} = \begin{cases} \dfrac{\lambda + p\mu}{p\mu}\pi_0, k = 0 \\ \left(\dfrac{\lambda + q\mu}{p\mu}\right)^k \dfrac{\lambda}{p\mu}\pi_0, k \geq 1 \end{cases} \quad (10)$$

When k=0, $P（L_q = 0）= \pi_0 + \pi_1 = \pi_0 + \dfrac{\lambda}{p\mu}\pi_0 = \dfrac{\lambda + p\mu}{p\mu}\pi_0$.

k≥1 时 $P（L_q = k）= \pi_{k+1} = (\dfrac{\lambda + q\mu}{p\mu})^k \dfrac{\lambda}{p\mu}\pi_0, k = 1,2,3,\cdots$

Under the steady state condition of the system, the distribution of W time waiting for the patient to wait is:

$$F_w(t) = \begin{cases} 0, t \leq 0 \\ \pi_0 + \dfrac{\lambda\pi_0}{p\mu - \lambda - q\mu}(1 - e^{-\mu(1-\frac{\lambda+q\mu}{-p\mu})t}), t>0 \end{cases} \quad (11)$$

W represents the waiting time of the customer in steady state, so

$$P(W = 0) = P(L = 0) = \pi \quad (12)$$

For any real number t>0, the distribution function of W is

$$F_w(t) = P(W = 0) + P（0<W<t） \quad (13)$$

$$P（0<W<t）= \sum_{k=1}^{\infty} P(0<W<t \mid L = k）P(L = k) = \sum_{k=1}^{\infty} P(0<W<t \mid L = k）\pi_k$$

$$= \sum_{k=1}^{\infty} \pi_k \int_0^t \frac{\mu^k x^{k-1}}{\Gamma(k)} e^{-\mu x} dx \quad (14)$$

$$\pi_k = \pi_0 (\frac{\lambda + q\mu}{p\mu})^{k-1} \frac{\lambda}{p\mu}, k = 1, \cdots, K, \cdots \quad (15)$$

Original= $\displaystyle\int_0^t \pi_0 \frac{\lambda}{p\mu} \sum_{k=1}^{\infty} \frac{\left(\mu x \cdot \dfrac{\lambda + q\mu}{p\mu}\right)^{k-1}}{(k-1)!} \mu e^{-\mu x} dx = \frac{\lambda}{p}\pi_0 \int_0^t e^{-\mu(1-\frac{\lambda+q\mu}{p\mu})x} dx$

$$= \frac{\lambda\pi_0}{p\mu - \lambda - q\mu}(1 - e^{-\mu(1-\frac{\lambda+q\mu}{p\mu})t}) \quad (16)$$

$$F(t)=\pi_0+\frac{\lambda\pi_0}{p\mu-\lambda-q\mu}(1-e^{-\mu(1-\frac{\lambda+q\mu}{p\mu})t})$$ is right. (17)

4.3.3System performance indicators

Assume that there are enough patients, that is, K is large enough:

(1) The average patient length of the system is:

$$E(L)=\frac{\lambda p\mu}{(p\mu-\lambda-q\mu)^2}\pi_0 \tag{18}$$

By the conclusion of Theorem 1:

$$E(L)=\sum_{k=0}^{\infty}k\pi_k \tag{19}$$

(2) The average patient waiting for the team leader is:

$$E(L_q)=\frac{\lambda+q\mu}{(p\mu-\lambda-q\mu)^2}\pi_0 \tag{20}$$

(3)After the patient arrives at the clinic, the probability of not waiting is:

$$\pi_0=1-\frac{\lambda}{(p-q)\mu} \tag{21}$$

(4)The probability that patients need to wait after arriving at the clinic is:

$$p=P(L\geq 1)=\sum_{k=1}^{\infty}\pi_k=\frac{\lambda\pi_0}{p\mu-\lambda-q\mu} \tag{22}$$

(5)After the patient arrives at the clinic, the average waiting time E(W) is:

$$E(W)=\frac{E(L_q)}{\lambda}=\frac{\lambda+q\mu}{\lambda(p\mu-\lambda-q\mu)^2}\pi_0 \tag{23}$$

Combined with the above data, consider the number of outpatients per day during the normal working days of the hospital. According to the number of outpatient doctors on the day, statistical software such as Matlab and Sas can be used to analyze the data. For example, analyze the utilization rate of the internal outpatient system on the day, the average vacancy rate of the system, and the average number of people per hour in the patient visit queue, the waiting time for waiting for medical treatment, and the probability of long queue time. For example, the patient queues for more than 1 hour. Probability and so on. According to the above analysis results, the number of outpatient doctors can be properly regulated. Under the premise of ensuring high utilization rate of the treatment system and low vacancy rate, the patient's queuing time can be reduced as much as possible, and the problem of crowded queuing can be solved, and the problem can be maximized. Rational use of hospital resources

4.4Establishment and Solution of Problem Four Mode

4.4.1Model establishment

The topic asks for the complex cooperation and competition between private hospitals and public hospitals, and proposes the best cooperation and competition strategy among many hospitals. This paper uses the maximization method to deeply study the cooperation and competition strategy between private hospitals and public hospitals. By studying the cooperative decision-making problem of resource sharing and utilization between private hospitals and public hospitals, a medical service product output decision model is established to compare private hospitals with public hospitals. The joint decision-making and the independent decision-making of the medical services market and their optimal decision-making output of medical service products.

4.4.2Symbiotic Synergy Decision Hypothesis

The medical resources (such as medical information, medical technology or medical equipment) of private hospitals can be used as the basis for medical treatment or resource sharing in public hospitals. Private hospitals and public hospitals can share resources or utilize related symbiotic systems. The production cost of private hospital units is c1 (including the cost to obtain medical shared resources and the cost of providing medical services to hospitals), and the rate of medical shared resources providing medical services to hospitals is β ($\beta>0$), from private The unit cost of transferring medical resources to hospitals from public hospitals is d1. In addition to considering the transfer of medical resources between different hospitals, it may also face the situation that hospitals over-utilize medical resources to pass them on to patients. The government is trying to curb this phenomenon. The cost of a fine imposed by the hospital. Public hospitals purchase medical shared resources from private hospitals at prices. The unit production cost of public hospitals is c2 (including the cost to obtain medical shared resources and the cost of providing medical services to hospitals), and the rate of public hospitals sharing resources for medical resources. δ ($\delta>0$). When the demand for private shared medical resources of public hospitals is less than that of private hospitals, the remaining private hospitals must bear the corresponding purchase costs; when public hospitals have more demand for medical shared resources of private hospitals than private hospitals, Insufficient public hospitals purchase other alternative medical resources at price d2.

It is assumed that the medical services provided by private hospitals and public hospitals are all price-elastic, and their prices and demand satisfy a linear relationship, $Qi=ai-biPi$ ($i=1, 2, a>0, b>0$). When $P3<0$, it means that private hospitals pay a certain fee to public hospitals to help public hospitals purchase medical resources. Let $-bi\leq P3\leq d2$, that is, the cost paid by private hospitals to public hospitals will not be higher than the cost of self-purchasing. The purchase price of public hospitals will not be higher than the price of alternative medical resources. Private hospitals and public hospitals have Complete information.

4.4.3Co-operation decision model

The model variables in this study are set as follows: π1 is the benefit function of private hospitals, the benefit function of π2 public hospitals; π is the benefit function of symbiosis between private hospitals and public hospitals, P1 is the price of medical services provided by private hospitals, and P2 is the public hospital. The price of medical services provided, Q1 provides the number of medical services provided by private hospitals, Q2 provides the number of medical services provided by public hospitals, Q1* is the optimal supply of private hospitals for symbiotic decision-making, and Q2* is the public hospital for symbiotic decision-making The optimal supply of medical services, Q^1 is the optimal supply of medical services when private hospitals make independent decisions, and Q^2 is the optimal supply of medical services when public hospitals make independent decisions, π* is private hospitals and public hospitals. The optimal benefit of the medical service system in symbiotic decision-making, π^ is the optimal benefit of the medical service market when private hospitals and public hospitals make independent decisions, Δπ is the benefit of the medical service market, ΔQ1 is the improvement of the supply of medical services in private hospitals, ΔQ2 For the improvement of the supply of medical services in public hospitals, ΔP1 is the price improvement of private hospital medical services, ΔP2 is a public hospital Spa services price improvement, ai, bi is the demand function coefficient (i = 1,2). Private hospitals and public hospitals take the maximum benefit as the decision-making goal. Therefore, the symbiotic decision-making model between private hospitals and public hospitals is:

$$\pi_1(Q_1) = P_1Q_1 + P_3\min(\delta Q_2, \beta Q_1) - c_1Q_1 - d_1\max(\beta Q_2 - \delta Q_1, 0) \quad (1)$$

$$\pi_2(Q_2) = P_2Q_2 + P_3\min(\delta Q_2, \beta Q_1) - c_2Q_2 - d_2\max(\beta Q_2 - \delta Q_1, 0) \quad (2)$$

$$\pi(Q_1, Q_2) = P_1Q_1 + P_2Q_2 - c_1Q_1 - c_2Q_2 - d_1\max(\beta Q_1 - \delta Q_2, 0) - d_2\max(\delta Q_2, \beta Q_1, 0) \quad (3)$$

$$P_1 = (a_1, Q_1)/b_1 \quad (4)$$

$$P_2 = (a_2, Q_2)/b_2 \quad (5)$$

When the medical shared resources supply exceeds the demand situation analysis.

According to the interest function of private hospitals, Equation 1, Equation 2 and Equation 3 can be rewritten as:

$$\pi_1(Q_1) = P_1Q_1 + P_3\delta Q_2 - c_1Q_1 - d_1(\beta Q_1 - \delta Q_2) \quad (6)$$

$$\pi_2(Q_2) = P_2Q_2 + P_3\delta Q_2 - c_2Q_2 \quad (7)$$

$$\pi_1(Q_1,Q_2)=P_1Q_1+P_2Q_2-c_1Q_1-d_1(\beta Q_1-\delta Q_2) \quad (8)$$

Private hospitals and public hospitals fully cooperate and make decisions. In the case of full cooperation between private hospitals and public hospitals, the amount and price of medical services provided by private hospitals and public hospitals are coordinated to make the benefits of the medical and health services market maximize.

Substituting Equation 4 and Formula 5 into Equation 8 can be obtained:

$$\pi_1(Q_1,Q_2)=(\frac{a_1}{b_1}-c_1-d_1\beta)Q_1+(\frac{a_2}{b_2}-c_2-d_1\delta)\ Q_2-\frac{Q_1^2}{b_1}-\frac{Q_2^2}{b_2} \quad (9)$$

When Q1 is fixed, the condition maximized by π, for any given Q1, is: $(\frac{a_2}{b_2}-c_2-d_1\delta)-\frac{2Q_2}{b_2}=0$。

$$Q_2^*(Q_1)=(a_2-b_2c_2+b_2d_1\delta)/2 \quad (10)$$

Substituting Equation 10 into Equation 8 to obtain the π value, and setting π=g(Q1), then

$$g(Q_1)=(\frac{a_1}{b_1}-c_1-d_1\beta)Q_1+(\frac{a_2}{b_2}-c_2-d_1\delta)Q_2^*(Q_1)-\frac{Q_1^2}{b_1}-\frac{(Q_2^*(Q_1))^2}{b_2} \quad (11)$$

When the condition of πmax is g(Q1), the first-order condition is zero, then Q1* can be obtained, namely:

$$Q_1^*=(a_1-b_1c_1-b_1d_1\beta)/2 \quad (12)$$

$$Q_2^*=(a_2-b_2c_2-b_2d_1\delta)/2 \quad (13)$$

Therefore, After cooperation between private hospitals and public hospitals, when the medical service market is optimal, the prices of the respective medical services are:

$$P_1^*=(a_1+b_1c_1+b_1d_1\beta)/2b_1 \quad (14)$$

$$P_2^*=(a_2+b_2c_2+b_2d_1\delta)/2b_2 \quad (15)$$

The formula 12, the formula 13, the formula 14, and the formula 15 are substituted into the formula 9 for calculation, and the optimal interests of the cooperative medical and health service system composed of the private hospital and the public hospital are:

$$\pi^*=(\frac{a_1}{b_1}-c_1-d_1\beta)(a_1-b_1c_1-b_1d_1\beta)/2+(\frac{a_2}{b_2}-c_2+d_1\delta)(a_2-b_2c_2-b_2d_1\delta)/2-\frac{(Q_1^*)^2}{b_1}-\frac{(Q_2^*)^2}{b_2} \quad (16)$$

Independent decision-making between private hospitals and public hospitals. Public hospitals are the first decision-makers, and the number of medical services provided by public hospitals is Q2. At this time, on the basis of the decision-making of public hospitals in private hospitals, the demand for medical shared resources provided by public hospitals to private hospitals is δQ2, and medical sharing is determined. The price of resources for P3 and the number of medical services provided by private hospitals is Q1.

In order to better explore the impact of private hospitals and public hospitals on the market benefits of medical services after independent decision-making, when private hospitals know that the demand for medical shared resources in public hospitals is δQ2, private hospitals make decisions on P3 and Q3 to make private Hospital benefits are maximized. Substituting Equation 4 and Formula 5 into Equation 6 and Equation 7 can be obtained:

$$\pi_1(Q_1) = (\frac{a1}{b1} - c1 - d1\beta)Q_1 - \frac{Q_1^2}{b_1} + (P_3 + d_1)\delta Q_2 \tag{17}$$

$$\pi_2(Q_2) = (\frac{a_2}{b_2} - P_3\delta - c_2)Q_2 - \frac{Q_2^2}{b_2} \tag{18}$$

Fixing Q1 in Equation 17 shows that π1(Q1) is an increasing function of P3, and private hospitals will maximize the price of medical shared resources to maximize their benefits. Because P3≤d2, the best medical treatment in private hospitals The shared resource price P^3 is always equal to d2, that is, P^3=d2. Substituting P^3=d2 into Equation 17, according to the condition of the maximum value of π1(Q1):

$$(\frac{a_1}{b_1} - c_1 - d_1\beta) - \frac{2Q_1}{b_1} = 0$$

The amount of π1 medical service that is derived from the private hospital is estimated as follows:

$$\hat{Q}_1 = (a_1 - b_1c_1 - b_1d_1\beta)/2 \tag{19}$$

Similarly, P^3=d2 is substituted into π2(Q2) (Equation 18), and according to the condition of π2(Q2) maximum value: $(\frac{a_2}{b_2} - c_2 - d_2\beta) - \frac{2Q2}{b_2} = 0$ 。

The amount of medical services that is pushed to the maximum in public hospitals is as follows:

$$\hat{Q}_2 = (a_2 - b_2c_2 - b_2d_2\delta)/2 \tag{20}$$

Therefore, the number of medical services provided by private hospitals and public hospitals under the independent decision-making conditions is optimal $(\hat{Q}_1, \hat{Q}_2)$. The market price of the corresponding medical service can be obtained by using the formulas 19 and 20 in the formula 4 and the formula 5:

$$\hat{P}_1 = (a_1 + b_1c_1 + b_1d_1\beta)/2b_1 \tag{21}$$

$$\hat{P}_2 = (a_2 + b_2c_2 + b_2d_1\delta)/2b_2 \tag{22}$$

The formula 19, the formula 20, the formula 21, and the formula 22 are substituted into the formula 9 for calculation. At this time, the interests of the private service hospital and the public hospital in the decision-making process are:

$$\hat{\pi} = \hat{\pi}_1(\hat{Q}_1, P_3) + \hat{\pi}_2(\hat{Q}_2) = (\frac{a_1}{b_1} - c_1 - d_1\beta)\hat{Q}_1 + (\frac{a_2}{b_2} - c_2 + d_2\beta)\hat{Q}_2 - \frac{\hat{Q}_1^2}{b_1} - \frac{\hat{Q}_2^2}{b_2} \tag{23}$$

Analysis of medical shared resource supply less than demand.If the supply of medical shared resources is less than the demand, then the benefit function 1 of private hospitals providing medical services is rewritten as:

$$\pi_1(Q_1) = P_1Q_1 + (P_3\beta - c_1)Q_1 \tag{24}$$

The utility function 2 of public hospitals providing medical services is rewritten as:

$$\pi_2(Q_2) = P_2Q_2 + (d_2\beta - P_3\beta)Q_1 - (c_2 - d_2\delta)Q_2 \tag{25}$$

The overall benefits of the medical services market consisting of private hospitals and public hospitals are:

$$\pi = P_1Q_1 + P_2Q_2 + (d_2\beta - c_1)Q_1 - (c_2 + d_2\delta)Q_2 \tag{26}$$

Private hospitals cooperate with public hospitals to make decision-making. In the case of full cooperation between private hospitals and public hospitals, the maximum amount of benefits is the basis for decision-making, and the supply and price decisions are made when providing medical services to the medical service market. Substituting Equation 4 and Equation 5 into Equation 26 yields:

$$\pi(Q_1, Q_2) = (\frac{a_1}{b_1} - c_1 - d_1\beta)Q_1 + (\frac{a_2}{b_2} - c_2 + d_2\delta)Q_2 - \frac{Q_1^1}{b_1} - \frac{Q_2^2}{b_2} \tag{27}$$

According to Equation 27, assuming that Q1 is fixed, the condition for maximizing by π is obtained: $(\frac{a_2}{b_2} - c_2 - d_2\delta) - \frac{2Q_2}{b_2} = 0$ 。

$$Q_2^*(Q_1) = (a_2 - b_2c_2 - b_2d_2\delta)/2 \tag{28}$$

Set $Q_2^*(Q_1)$ correspond to a π value of $g(Q_1)$, then:

$$g(Q_1) = (\frac{a_1}{b_1} - c_1 + d_2\beta)Q_1 + (\frac{a_2}{b_2} - c_2 - d_2\delta)Q_2^*(Q_1) - \frac{Q_1^2}{b_1} - \frac{(Q_2^*(Q_1))^2}{b_2} \tag{29}$$

When πmax exists, it is obtained $Q_1^* = (a_1 - b_1c_1 - b_1d_2\beta)/2$ 。

Therefore, when private hospitals and public hospitals cooperate fully, the number of their own optimal medical services is:

$$(Q_1^*, Q_2^*) = ((a_1 - b_1c_1 + b_1d_2\beta)/2, (a_2 - b_2c_2 + b_2d_2\delta)/2) \qquad (30)$$

Substituting the formula 30 into the formula 4 and the formula 5, the optimal price at this time is obtained as follows:

$$(P_1^*, P_2^*) = ((a_1 + b_1c_1 + b_1d_2\beta)/2b_1, (a_2 + b_2c_2 - b_2d_2\delta)/2b_2) \qquad (31)$$

Therefore, the formula 30 and the formula 31 are substituted into the total benefit function of the medical service market, and the following are obtained:

$$\pi^*(Q_1, Q_2) = (\frac{a_1}{b_1} - c_1 + d_2\beta)Q_1^* + (\frac{a_2}{b_2} - c_2 - d_2\delta)Q_2^* - \frac{Q_1^*}{b_1} - \frac{(Q_2^*)^2}{b_2} \qquad (32)$$

Private hospitals and public hospitals do not cooperate independently to make decisions. Assume that the public hospital decides first and decides the quantity Q2 of medical services provided by public hospitals. At this time, the demand for medical shared resources is δQ2. Private hospitals follow the decision-makers. Based on the above situation, the price of medical shared resources P3 is determined. The number of medical services provided by private hospitals is Q1, which determines the number of medical shared resources provided by private hospitals is βQ1. The formula 25 and the formula 5 can be obtained:

$$\pi_1(Q_1) = (\frac{a_1}{b_1} + P_3\beta - c_1)Q_1 - \frac{Q_1^2}{b_1} \qquad (33)$$

$$\pi_2(Q_2) = (d_2 - P_3)\beta Q_1 + (\frac{a_2}{b_2} - c_2 - d_2\delta)Q_2 - \frac{Q_2^2}{b_2} \qquad (34)$$

It can be seen from Equation 33 and Equation 34 that π1 is an increasing function of P3. Private hospitals will increase the price of medical shared resources to achieve their own optimality. Because P3≤d2, the optimal medical shared resource price of private hospitals Constant is equal to d2, ie P^3=d2. Substituting P^3=d2 into Equation 33, the first-order condition maximized by π1 is obtained:

$$(\frac{a_1}{b_1} + d_2 - c_1)\frac{Q_1}{b_1}, \hat{Q}_1 = (a_1 + b_1d_2 - b_1c_1)/2 \qquad (35)$$

Substituting P^3=d2 and Equation 35 into Equation 34 yields:

$$\pi_2(Q_2, d_2, \hat{Q}_1) = (\frac{a_2}{b_2} - c_2 - d_2\delta)Q_2 - \frac{Q_2^2}{b_2} \qquad (36)$$

According to the first-order conditions for public hospitals to maximize their own interests:

$$\hat{Q}_2 = (a_2 - b_2c_2 - b_2d_2\delta)/2 \qquad (37)$$

In this way, the optimal medical service supply and the corresponding market price when private hospitals and public hospitals make independent decisions are:

$$(\hat{Q}_1,\hat{Q}_2)=((a_1+b_1P_3-b_1c_1)/2,(a_2-b_2c_2-b_2d_2\delta)/2) \tag{38}$$

$$\hat{P}_1,\hat{P}_2)=((a_1-b_1P_3+b_1c_1)/2,(a_2+b_2c_2+b_2d_2\delta)/2) \tag{39}$$

Therefore, independent decision-making between private hospitals and public hospitals brings benefits to the medical services market:

$$\hat{\pi}=\hat{\pi}_1(\hat{Q}_1,\hat{P}_3)+\hat{\pi}_2(\hat{Q}_2)=(\frac{a_1}{b_1}+d_2\beta-c_1)\hat{Q}_1+(\frac{a_2}{b_2}-d_2\delta-c_2)-\frac{\hat{Q}_1^2}{b_1}-\frac{\hat{Q}_2^2}{b_2} \tag{40}$$

Co-decision analysis of symbiosis between private hospitals and public hospitals.A comparison of strategies for the supply of medical shared resources greater than demand. Compared with the independent decision-making of private hospitals and public hospitals, when the two types of hospitals co-decisively make decisions in the medical service market, the benefits of the entire medical service market are improved. The total benefits and non-cooperative independent decisions of the medical service market are fully information cooperation. When the change in the total benefit of the medical services market is expressed, namely:

$$\Delta\pi=(\frac{a_1}{b_1}-c_1-d_1\beta)Q_1^*+(\frac{a_2}{b_2}-c_2+d_2\delta)Q_2^*-\frac{(Q_1^*)}{b_1}-\frac{(Q_2^*)^2}{b_2}-(\frac{a_1}{b_1}-c_1-d_1\beta)\hat{Q}_1$$

$$-(\frac{a_2}{b_2}-c_2+d_2\delta)\hat{Q}_2-\frac{\hat{Q}_1^2}{b_1}+\frac{\hat{Q}_2^2}{b_1} \tag{41}$$

$$\Delta\pi=b_2(d_1+d_2)^2\delta^{\cdot 2}/4\geq 0 \tag{42}$$

In the medical service market, the increase in the supply of medical services provided by private hospitals is $\Delta Q_1=0$, the increase in the supply of medical services provided by public hospitals is $\Delta Q_2=b_2\delta(d_1+d_2)/2\geq 0$, and the increase in the price of medical services provided by private hospitals and public hospitals is $\Delta P_1=0$ 、 $\Delta P_2=-\delta(d_1+d_2)/2\leq 0$.

In summary, when the supply of medical shared resources exceeds demand, the synergy between private and public hospitals can bring the total benefits of the additional medical services market. At the same time, the supply of medical services in private hospitals remains unchanged, and the increase in the supply of medical services in public hospitals is $b_2\delta(d_1+d_2)/2$. The prices of private hospitals have not changed, and the prices of public hospitals have declined due to the increase in the number of medical services. It can be seen that the collaborative decision-making between the two is superior to the independent decision-making; in this case, the realization of the cooperation between private hospitals and public hospitals depends on the efforts of public hospitals. Whether the purpose of cooperation can be achieved depends on the incentives of private hospitals for public hospitals.

The comparison of the medical shared resources supply is less than the demand. Compared with independent decision-making and collaborative decision-making in private hospitals and public hospitals, the overall benefit improvement effect of the medical service market is mainly reflected in the comparison between the total benefits in the medical service market under independent decision-making and the total benefits in the medical service market under collaborative decision-making:

$$\Delta\pi = \pi^*(Q_1, Q_2) - \hat{\pi} = (\frac{a_1}{b_1} - c_1 + d_2\beta)Q_1^* + (\frac{a_2}{b_2} - c_2 - d_2\delta)Q_2^* - \frac{(Q_1^*)^2}{b_1} - \frac{(Q_2^*)^2}{b_2}$$

$$-(\frac{a_1}{b_1} - c_1 + d_2\beta)\hat{Q}_1 - (\frac{a_2}{b_2} - c_2 - d_2\delta) + \frac{\hat{Q}_1^2}{b_1} + \frac{\hat{Q}_2^2}{b_2} \tag{43}$$

Correspondingly, the supply increment and price increase of medical services provided by private hospitals and public hospitals are:

$$\Delta Q_1 = 0, \Delta Q_2 = 0, \Delta P_1 = 0, \Delta P_2 = 0 \tag{44}$$

According to the above analysis, compared with the independent decision-making of private hospitals and public hospitals, the coordinated decision-making between the two cannot bring additional benefits to the medical service market, and the quantity and price of medical services provided by private hospitals and public hospitals remain unchanged. Therefore, private hospitals Collaboration with public hospitals has no impact on the overall benefits of the health care market.

4.4.4Best Cooperation and Competition Strategy Proposal

First, build an information collaboration platform

According to the particularity of the medical service market, the information collaboration platform for private hospitals and public hospitals will be built, so that private hospitals and public hospitals can fully utilize their own advantages and actively cooperate to meet the multi-level medical service needs of patients. Based on the smooth flow of information, build an information sharing platform, rationally utilize the model of medical shared resources, and exert synergy to maximize the total benefits of the medical service market.

Second, the formation of an effective synergy mechanism

Under the guidance of the national medical and health policy, private hospitals and public hospitals will be coordinated, from the investment of funds, the cultivation of medical technology talents, the layout of hospitals, and the design of scales, so that private hospitals and public hospitals can fully utilize them. efficacy.

Third, establish a collaborative performance evaluation system

The evaluation of medical service providers is different from the performance evaluation of general enterprises. It is not to focus on the supply of medical services in private hospitals or public hospitals, but on the medical service market that will consist of private hospitals and public hospitals. The evaluation indicators should be time-sensitive, and should meet the patient's demand for multi-level medical services, and solve the problem that residents are “highly expensive and difficult to see”.

4.5Suggestions

(1)Through the establishment of grey clustering model to classify the local public and private hospitals, which are close to each other in a city, the hospitals belong to the same category are "married" to establish a community and promote cooperation, so as to explore a better development direction. We can set up a medical association expert workshop between public and private hospitals, open a two-way referral green channel, train medical staff at different levels for private hospitals, and send experts to carry out regular ward rounds for severe inpatients, so as to ensure medical quality and safety. For example, Renji Hospital Affiliated to Shanghai Jiaotong University Medical College signed a contract with Shanghai Tongkang hospital to build a Medical Association. This is the first Medical Association jointly built by the public top three hospitals and private hospitals in Shanghai. It provides safe, convenient and high-quality medical services for the surrounding residents, especially the elderly patients, and jointly explores the mode of medical and nursing integration based on local conditions

(2) the government should create a policy environment of fair competition to help form a healthy competition system in the medical market and improve the service efficiency of the health system. In terms of laws and policies, we should maintain the development environment of different ownership medical institutions, establish the idea of market leading and government guiding, and create a good environment for fair competition. Only when the public and private hospitals are in a quite competitive position, can the market competition be more sufficient, and also fundamentally promote the balanced development and common improvement of the two types of hospitals, so as to achieve the development and improvement of medical and health services. Due to the late start of private hospitals, weak foundation, technical level, medical facilities and other aspects are difficult to compete with public hospitals, so in particular, the industry access, price policy, business guidance, medical insurance fixed point, tax collection, policy coordination and other aspects need further support from the government.

(3)Build an information collaboration platform and form an effective collaboration mechanism .Private hospitals and public hospitals need to build an information platform to form a vertical medical service system with classified configuration and two-way referral, so that private hospitals and public hospitals can give full play to their own advantages and actively cooperate to meet the multi-level medical service needs of patients. In order to promote the sharing of resources and medical information and improve the level of medical services at all levels, we should realize the horizontal integration of medical resources with mutual recognition of results and complementary advantages. Rational utilization of medical shared resources model, giving full play to collaborative efficiency, and maximizing the total benefits of medical service market.

5.Conclusion

ARIMA prediction model, which can predict the elderly population in China from 2011 to 2050, has a high fitting accuracy. The population prediction model chooses the age migration model, which is easy to understand in theory, rigorous in principle, simple in method and with high accuracy.In addition, the grey GM (1,1) prediction equations all achieve $C < 0.35$, $P = 1$, and the fitting effect is matched

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Appendix

```
y=input('Please enter data');
n=length(y);
yy=ones(n,1);
yy(1)=y(1);
for i=2:n
    yy(i)=yy(i-1)+y(i);
end
B=ones(n-1,2);
for i=1:(n-1)
    B(i,1)=-(yy(i)+yy(i+1))/2;
    B(i,2)=1;
end
BT=B';
for j=1:(n-1)
    YN(j)=y(j+1);
end
YN=YN';
```

```
A=inv(BT*B)*BT*YN;
a=A(1);
u=A(2);
t=u/a;
t_test=input('Enter the number of predictions you need to make');
i=1:t_test+n;
yys(i+1)=(y(1)-t).*exp(-a.*i)+t;
yys(1)=y(1);
for j=n+t_test:-1:2
    ys(j)=yys(j)-yys(j-1);
end
x=1:n;
xs=2:n+t_test;
yn=ys(2:n+t_test);
plot(x,y,'^r',xs,yn,'*-b');
det=0;

sum1=0;
sumpe=0;
for i=1:n
    sumpe=sumpe+y(i);
end
pe=sumpe/n;
for i=1:n;
    sum1=sum1+(y(i)-pe).^2;
end
s1=sqrt(sum1/n);
sumce=0;
for i=2:n
    sumce=sumce+(y(i)-yn(i));
end
ce=sumce/(n-1);
sum2=0;
for i=2:n;
    sum2=sum2+(y(i)-yn(i)-ce).^2;
end
s2=sqrt(sum2/(n-1));
c=(s2)/(s1);
disp(['The posterior difference ratio is：',num2str(c)]);
if c <0.35
     disp('System prediction accuracy is good')
else if c<0.5
     disp('The system prediction accuracy is qualified')
   else if c<0.65
```

```
                disp('The system predicts with barely adequate accuracy')
            else
                disp('System prediction accuracy is not up to standard')
            end
        end
    end

    disp(['The next fitting value is：',num2str(ys(n+1))]);
    disp(['Then the fitting value is：',num2str(ys(n+2))]);
    s = 24; % cycle is 24
    x = xixi';% initial data entry
    n = 24; the number of % forecasts
    M1 = length(x); % of the original data
    For i = s+1:m1;
      y(i-s) = x(i) - x(i-s);% performs periodic differential transformation
    End
    w = diff(y); % eliminates trending difference operations
    M2 = length(2);
    For i = 0:6
      For j = 0:6
        If i == 0 & j == 0
          Continue
        Elseif i == 0
            ToEstMd = arima('MALags',1:j,'Constant',0); % specifies the structure of
the model
        Elseif j == 0
            ToEstMd = arima('ARLags',1:i,'Constant',0); % specifies the structure of
the model
        Else
            ToEstMd = arima('ARLags',1:i,'MALags',1:j,'Constant',0); % specifies
the structure of the model
        End
        k = k + 1;
        R(k) = i;
        M(k) = j;
        [EstMd, EstParamCov, LogL, info] = estimate(ToEstMd,w');% model fit
          numParams = sum(any(EstParamCov));% calculates the number of fitting
parameters
        [aic(k), bic(k)] = aicbic(LogL, numParams, m2);
      End
    End
    Fprintf('R, M, AIC, BIC corresponding value is as follows \n%f'); % shows the
calculation result
    Check = [R',M',aic',bic']
```

x = xixi';

ToEstMd = arima('ARLags',1:5,'MALags',1:2,'Constant',0);% specifies the structure of the model

[EstMd, EstParamCov, LogL, info] = estimate(ToEstMd,w');% model fit

w_Forecast = forecast(EstMd,n,'Y0',w');

Yhat = y(end) + cumsum(w_Forecast); % first-order difference restore value

For j = 1:n

x(m1 + j) = yhat(j) + x(m1+j-s); predicted value of %x

End

x(m1+1:end)

M / M / 1 Queuing System:

Clear;

Clc;

Alpha=0.1; % confidence level

Gama=0.1; % relative accuracy

Beta=0.1;

Lambda=0.2; % arrival rate Lambda

Mu=0.25; % service rate Mu

Time=50; % single-return simulations

% sequential method implementation

All_vector=Func(Lambda,Mu); the vector returned by the % function

For i=2:time

All_vector=[All_vector;Func(Lambda,Mu)];

End

Vect=sum(All_vector)/time;% of each column is summed to average

% The average waiting queue in the system below is used as the X for each simulation to evaluate the running results.

S=sum((All_vector(3)-vect(3)).*(All_vector(3)-vect(3)))/(ti

Me-1);% followed by S unchanged

Betan=sqrt(S/time)*tinv(1-Alpha/2,time-1);

Gaman=Betan/vect(3);

While(Gaman>=Gama) %Betan>=Beta||

Time=time+1;

All_vector=[All_vector;Func(Lambda,Mu)];

Vect=sum(All_vector)/time;

S=sum((All_vector(3)-vect(3)).*(All_vector(3)-vect(3)))/(ti

Me-1);% followed by S unchanged

Betan=sqrt(S/time)*tinv(1-Alpha/2,time-1);

Gaman=Betan/vect(3);

End

Time

Disp (['The lower bound of the confidence interval of the average waiting for the captain in the system

=',num2str(vect(3)-Betan)]);

Disp (['The upper bound of the confidence interval of the average waiting for the captain in the system

=',num2str(vect(3)+Betan)]);

Function [vector] = Func(Lambda,Mu)

% single queuing simulation, sample number CusTotal

CusTotal=10000; % simulation customer total %=input('Please enter simulation customer

Total CusTotal=');

Cus_Arrive=zeros(1,CusTotal);% arrival time

Cus_Leave=zeros(1,CusTotal);% departure time

IntervaCus_Arrive=-log(rand(1,CusTotal))/Lambda;% arrival time

Separate

Cus_Arrive=cumsum(IntervaCus_Arrive); % per column

Accumulate, forming the initial time; if there is only one line, the line is superimposed backwards

Interval_Serve=-log(rand(1,CusTotal))/Mu; % service interval

% prepare for event scheduling

Cus_Leave(1)=Cus_Arrive(1)+Interval_Serve(1);% customer departure time

For i=2:CusTotal

If Cus_Leave(i-1)<Cus_Arrive(i)

Cus_Leave(i)=Cus_Arrive(i)+Interval_Serve(i);

Else

Cus_Leave(i)=Cus_Leave(i-1)+Interval_Serve(i);

End

End

Cus_Wait=Cus_Leave-Cus_Arrive; % waiting time for each customer in the system

%mean: If it is a matrix of n*m, mean averaging each column separately; when n=1,

Average one line

Cus_Wait_avg=mean(Cus_Wait); % average waiting time

Cus_Queue=Cus_Wait-Interval_Serve;% of each customer's queue time in the system

Cus_Queue_avg=mean(Cus_Queue); % average queuing time

%TimePoint system scheduling time

TimePoint=[Cus_Arrive,Cus_Leave];% of customers in the system change over time

TimePoint=sort(TimePoint);

CusNum=zeros(size(TimePoint));

Temp=2; % points to the event table

CusNum(1)=1;

% statistic queuing people in dt time - event scheduling method

% is up to i, how many events are there?

For i=2:length(TimePoint)
%The end time of the next event is later than the end time of the previous event
% so as long as the first temp-1 event does not end, the following event reaches the time of occurrence
(Event simulation clock <= system simulation clock), must be added to CusNum count
If
(temp<=length(Cus_Arrive))&&(TimePoint(i)==Cus_Arrive(temp)
)
CusNum (i) = CusNum (i-1) +1;
temp = temp + 1;
Else
CusNum(i)=CusNum(i-1)-1;
End
End
Average number of customers in the % system
Time_interval = zeros (size (TimePoint));
Time_interval (1) = Cus_Arrive (1);
For i=2:length(TimePoint)
Time_interval(i)=TimePoint(i)-TimePoint(i-1);
End
%Draw a picture here.i The number of events counted in the time period *(i-1). Similar to
Post integration
CusNum_fromStart=[0 CusNum];
Cus_Wait_CusNum_avg=sum(CusNum_fromStart.*[Time_interval
0])/TimePoint(end);
% system average waiting for captain
QueLength=zeros(size(CusNum));
For i=1:length(CusNum)
If CusNum(i)>=2
% also has a wait event (satisfaction event simulation clock <= system simulation clock, but no
Health)
QueLength(i)=CusNum(i)-1;
Else
QueLength(i)=0;
End
End
Cus_Wait_Queue_avg=sum([0 QueLength].*[Time_interval
0])/TimePoint(end);
% simulation value compared with theoretical value
Row=Lambda/Mu;
QueLength_avg=row*row/(1-row);%Q

```
CusNum_avg=row/(1-row);%L
Queue_avg=QueLength_avg/Lambda;%d
Wait_avg=CusNum_avg/Lambda;%w
% returned vector
Vector=[Cus_Queue_avg,Cus_Wait_avg,Cus_Wait_Queue_avg,Cus_W
ait_CusNum_avg];
end
```