Team Control Number

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Problem Chosen

B

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ShuWei Cup
Summary Sheet

Summary

UAV shooting is becoming popular today because it shoots the object from multi-angle via flying over it. This paper mainly establishes geometric models to analyze the changes in height and angle over time to obtain the best practice. The results are reliable and practical since it is verified by a large numbers of simulation experiments.

Issue 1. Firstly, we analyze imaging principle and the relationship between focal length and visual angle, create geometric model, solve the oblique cone equation of the visual field. Then establish an evaluation criterion of whether it is at the best shooting height according to the target position. By combining the oblique cone equation and the ground equation, the elliptical field of view of UAV on the ground and optimal height are obtained. Finally, combined with MATLAB simulation, get the relationship between shooting angle and shooting height in the best practice.

Issue 2. when the UAV is flying towards the target object, the time that the object stays in the field of vision depends on the length of the long axis of the elliptical field of vision, the larger the angle is, the longer the long axis and the stay time are, and the field of view may become too large. Combined with mathematics and figures, we create geometric model of the elliptical field of view, figure out relationship between the inclination angle and dwell time/the proportion of the target area respectively, with MATLAB simulation, confirm the best shooting angle range is 35-55 and 125-145 finally.

Issue 3, the UAV needs to adjust its shooting angle and shooting height while following the moving object. Firstly, we find the maximum allowable shooting angle (ensure the target occupies a certain proportion in the vision field), then explore the angle and height of the UAV which need to be adjusted when the object moves forward the same distance in different status. Finally, get the experimental results with the specific situation and MATLAB simulation.

Issue 4, the simulated scene is constructed first (Target is a 4m- radius cylinder and the horizontal distance between the drone and the target is 40m). The UAV adjusts the angle and height while flying towards the target. By analyzing the geometric relationship, the equation of position, rotation angle and height with time is established. Finally, get the data with MATLAB simulation.

Key word: **MATLAB simulation   UAV shooting   Oblique cone equation**

Content

1. Background

Aerial photographing with drones can give us a broader perspective and show the overall setting. Shooting with a drone often takes a certain height from the ground to form a high-pitched effect. This effect can bring some advantages on the screen,enabling a wider range of shots and a wider field of view

However, for the first-time drone operator, it is difficult to master the techniques of flying height, flying speed and the shooting angle. If the flying height of the drone is too high, the range collected by the camera will be relatively large, but the pixel points per unit area will be less; if the flying height of the drone is too low, the range will be relatively small, the number of pixels in the area will increase and the accuracy of the collected information will be improved; If the flying speed of the drone is too fast, the number of frames taken by the camera will be reduced per unit time, and the difficulty of image analysis will be increased. The size of the shooting angle is related to the effect of image shooting. How to use the drone to take a satisfactory picture is one of the most urgent problems to be solved.

Task1: Assume that the drone is not affected by external factors such as wind direction and humidity during flight, and the flight speed and shooting angle are constant, and the drone performs aerial photographing in a straight-line flight mode for an area with a certain width. What is the height range of the flying drone performing this aerial photographing and propose the best aerial photographing height at the specified shooting angle.

Task2: Assume that the UAV's flight altitude is fixed and the flight speed is constant and the UAV needs to perform more accurate image acquisition on the actual ground conditions and please calculate the optimal range of the UAV shooting angle.

Task3: Assume that the flying height of the drone is fixed and the flying speed is constant and the drone is collecting the trajectory of a certain moving object on the ground. Please calculate the changing pattern of the UAV shooting angle.

Task4: If the drone encounters obstacles such as signal towers, telephone poles,houses, etc. during its flight, please calculate the optimal flight speed, flight altitudeand the shooting angle, and provide the relevant data simulation experiment.

2. Symbolic description and model hypothesis

2.1 Symbol description

symbol	significance
h	Flight altitude
α	Camera rotation angle
θ	Drone perspective
d_s	Horizontal distance

d	distance

2.2 Assumption

1.The UAV is always flying towards the target.
2.The UAV of the uav is not disturbed by airflow or wind.
3.The UAV travels at a constant speed as it goes straight.

3. The model establishment and solution of task1

3.1 UAV shooting angle

Looking at the relevant information, we know that the field of view of the camera depends mainly on the maximum shooting angle, and the maximum shooting angle depends mainly on the focal length of the camera and the film (photosensitive element size). Firstly, the maximum shooting angle of the drone is analyzed:

Looking at the relevant information, we understand that ordinary cameras generally follow the principle of optical imaging:The green line in the figure passes through the center of the lens, simplifying the whole process, and we will get the geometric relationship diagram shown below:
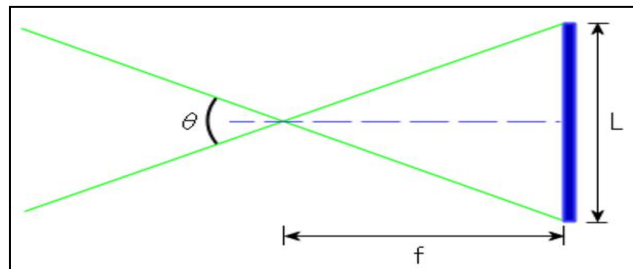

Figure 4 Principle of convex lens imaging(d)

Therefore, the equations can be listed: $\tan\dfrac{\theta}{2} = \dfrac{L}{2} f$

Solve to get: $\theta = 2\arctan(\dfrac{L}{2f})$ For the photosensitive element, L Can be approximated as the largest size, taking rectangular photosensitive elements as an example. L Should be the diagonal length of the rectangle.Browse the relevant parameters of mainstream cameras on the market, L Generally 5mm to 8mm.

Will L After considering the constant, you can get the relationship between the angle of view and the focal length as shown in the following figure:
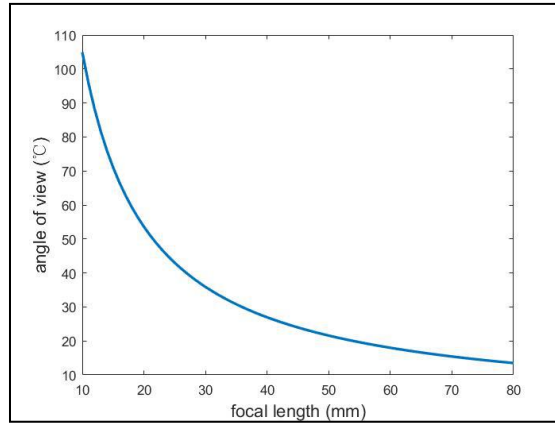
Figure 5 The relationship between theta and focal length

3.2 UAV shooting field of view modeling

In order to better display the shooting situation of the drone, a simple drawing of the drone is obtained, and a simplified model of the drone as shown in the following figure is obtained:
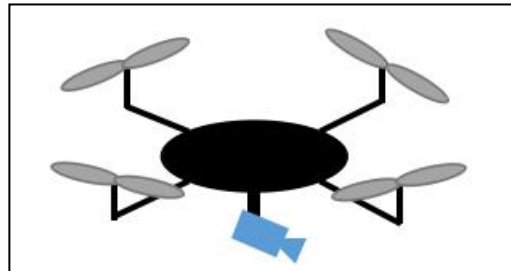

Figure 6 Uav diagram

The camera under the drone can be freely rotated along the end of the UAV bracket, only considering that the drone is moving toward the object, and the corner is defined as the angle with the direction of motion. α , the range is 0-180 degrees.

Figure 7 Shooting angle
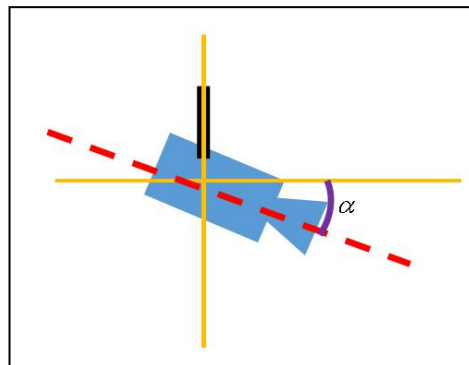
In the above, we mentioned that the camera has the largest viewing angle, so the scope of the drone can be seen as a cone during the shooting process.Below, we solve this conical surface.

First, you need to treat the camera's main axis as a straight line and solve its analytic formula because it has a corner. α And must pass the origin of the coordinate axis, so the analytical

formula of the line can be written as

$$l_1 : y = -\tan \alpha \cdot x \qquad (1.1)$$

Equipped in l_1 The conical surface on the upper can be regarded as l_1 Angle θ Straight

line l_2 Winding around l_1 Rotate one week to get.Next, we mathematically model the oblique cone

and establish a spatial coordinate system as shown below.
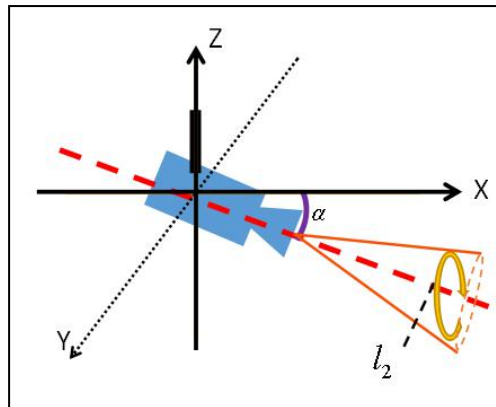

Figure 8 Field of View(a)

Get a little on the cone $A(x, y, z)$, the point and the axis l_1 The distance can be expressed by

the following distance formula:

$$d = \frac{|Ax + By + Cz + D|}{\sqrt{A^2 + B^2 + C^2}} \qquad (1.2)$$

In the cone, as shown in the following figure, the following equation can be established:
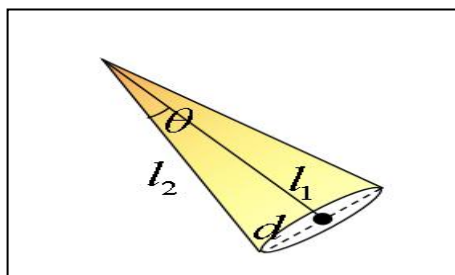
$$d = l_2 \sin \theta \qquad (1.3)$$

Figure 9 Analyze of field view(b)

Then in the oblique cone shown in the figure above, l_2 The length can be calculated from the

lens and the coordinates of the point, (the position of the lens is known), and the equations 1.2 and
1.3 can be used to obtain the equation of the oblique cone.

3.3 Solving the height of a plane object or a low-altitude object

When a drone photographs a plane object (such as the ground), its oblique cone will inevitably form an elliptical intersection with the photographed plane. Considering that the drone advances toward the photographed object, its imaged area will inevitably make a rectangle.Since the size of the camera itself is small, it is negligible relative to the height of the flight and the size of the object, so we simplified the model, and the origin of the established Cartesian coordinate system is set at the camera lens, as shown below.
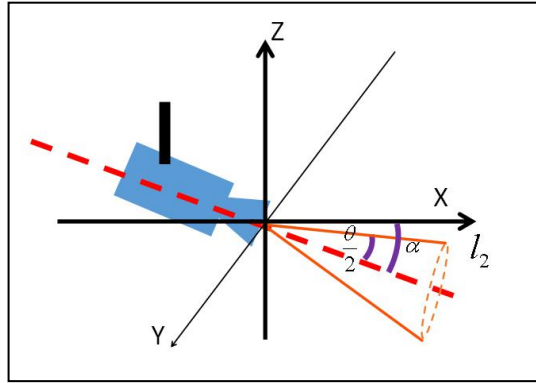

Figure 10 Analyze of field of view(c)

at this time, l_2 The line resolution is:

$$z = -\tan(\alpha - \frac{\theta}{2})x \qquad (1.4)$$

axis l_1 The analytical form is:

$$\tan \alpha \cdot x + z = 0 \qquad (1.5)$$

At this point, the coordinates of any point on the conical surface are $A(x, y, z)$, combined with the formula 1.1 and its straight line l_1 The distance is:

$$d = \frac{|\tan \alpha \cdot x + z|}{\sqrt{\tan^2 \alpha + 1}} \qquad (1.6)$$

Combined with Equation 1.2, the distance can also be written as:

$$d = \sin \frac{\theta}{2} \sqrt{x^2 + y^2 + z^2} \qquad (1.7)$$

Simultaneous Equation 1.5 1.6 has:

$$\frac{|\tan \alpha \cdot x + z|}{\sqrt{\tan^2 \alpha + 1}} = \sin \frac{\theta}{2} \sqrt{x^2 + y^2 + z^2}$$

Drone flight has a certain height h, so the above formula should be written as

$$\frac{|\tan\alpha \cdot x + (z-h)|}{\sqrt{\tan^2\alpha + 1}} = \sin\frac{\theta}{2}\sqrt{x^2 + y^2 + (z-h)^2} \tag{1.8}$$

In the above, we assume that the height of the object being photographed is negligible, then the field of view should be within the intersection of the conical surface and the xoy plane formed by Equation 1.8, so solving the field of view only requires $z = 0$, get the following elliptic equation:

$$\frac{|\tan\alpha \cdot x - h|}{\sqrt{\tan^2\alpha + 1}} = \sin\frac{\theta}{2}\sqrt{x^2 + y^2 + h^2} \tag{1.9}$$

Select the appropriate parameters and use matlab to draw the oblique conical field of view, as shown below:
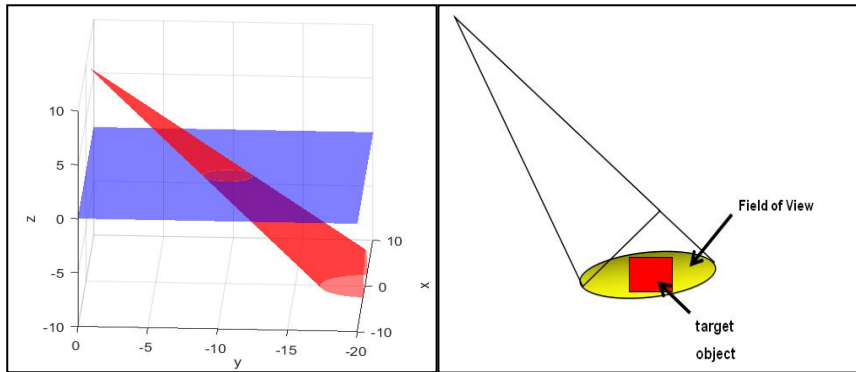

Figure 11-12 Oblique conic field of view

After the cone is tangent to the xoy plane, it will inevitably form the intersection of the ellipse.

First, we need to judge whether the object is completely inside the ellipse. The intersection of the xoy plane fields of view will write the obtained elliptic equation as follows:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Take $f(x,y) = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Give a judgment method for two situations:

1. If the object is a simple planar figure, take the coordinates of the feature points (four vertices if the rectangle is selected) $f(x,y)$ Judge $f(x,y)$ With a size of 1, if $f(x,y) < 1$, then the point inside the ellipse.

2. If the object is a complex planar figure, the Monte Carlo statistical method is used to randomly select a point with a certain number of object edges according to the size of the object, and substitute the elliptic equation to judge the same as above. $f(x,y)$ With a size of 1.

The above process can only determine whether the object can be completely captured into the picture. There is no limit to the size of the object in the picture. In order to obtain the best picture, refer to the relevant information, we have chosen the method of the largest size to judge the object in the picture. Is the space occupied by it appropriate?
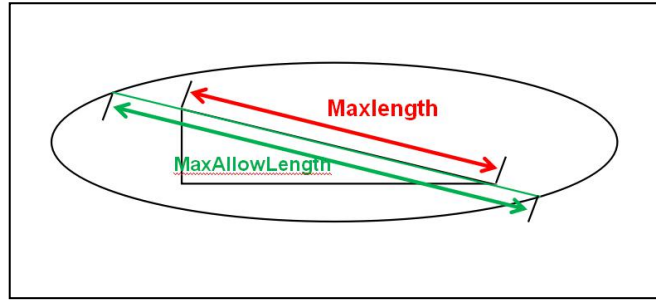
Figure 13 Maximum length judgment method

Any object always has a maximum length L_m In its direction, the ellipse has its maximum

allowable length L_a , if defined $0.8 < \dfrac{L_m}{L_a} < 0.85$, said it is in the best position.For the actual

situation, L_m There are always accurate or estimated values.Here, firstly, the analytical formula of

the straight line needs to be calculated, and the analytical equation and the elliptic equation are combined to obtain the coordinates of the two intersection points. The two-point coordinate formula can be used to obtain the size.

Set the analytical formula of the line whose maximum length is $l_m : y = k_m x + b_m$ The

elliptic equation obtained can be written as: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, using the elimination method to

eliminate the variable y ,get $\dfrac{x^2}{a^2} + \dfrac{(k_m x + b_m)^2}{b^2} = 1$ The line must have two intersections with the

ellipse. Solving the equation group yields the abscissa of the two points intersected. $x_1, x2$,

substituting it into the equation can get the coordinates of two points $(x_1, y_1), (x_2, y_2)$, using the

distance formula can be obtained

$$L_a = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad (1.10)$$

Defined here $\dfrac{L_m}{L_a} = 0.8$, considered to be in the best position.

3.4 Simulation

In this simulation, the size of the object to be photographed is 50m*50m, and the camera with a focal length of 30mm is selected, and the angle of view is 30 degrees.The center of the square object is always on the camera line of sight, adjusting the camera height h To the best

height h_b To explore the corner α With optimal height h_b Relationship.When $\alpha < \dfrac{\pi}{4}$ or $\alpha > \dfrac{3\pi}{4}$ The area of the ellipse formed by the cone and the xoy plane is too large. Therefore, in this simulation, only the consideration is considered. $\dfrac{\pi}{4} < \alpha < \dfrac{3\pi}{4}$ Case.Use matlab programming to simulate the whole shooting process, select some simulation experiment results, as shown below
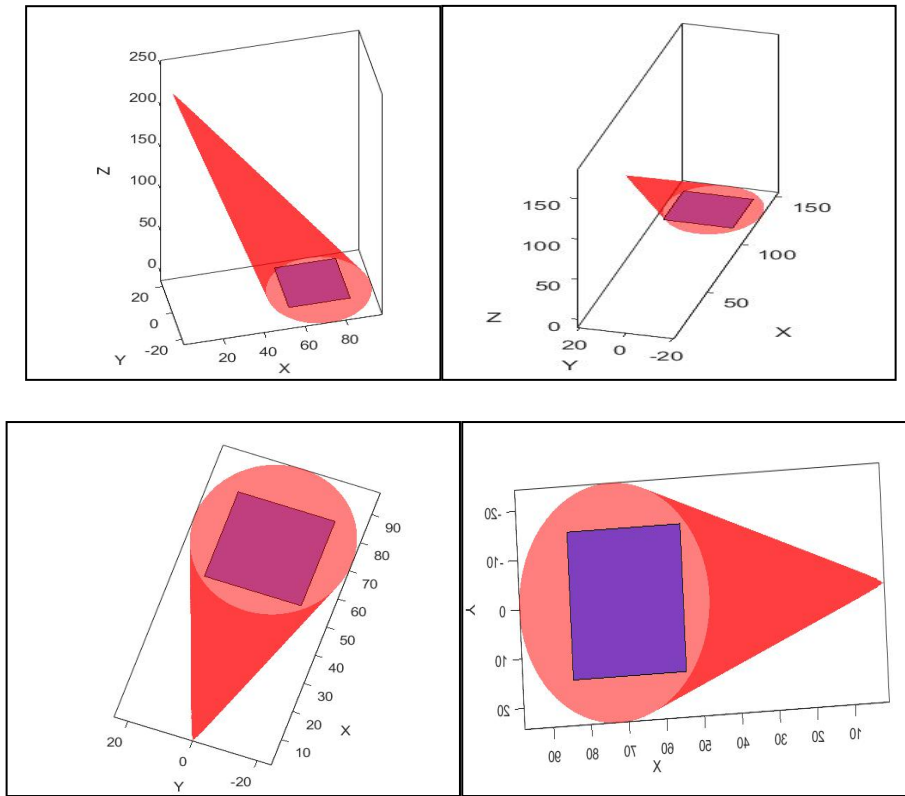

Figure 14 Simulation results(a-d)

The target is always in line of sight, and the simulation results are good. Finally, the relationship between the corner and the height is drawn when the best shot is taken.
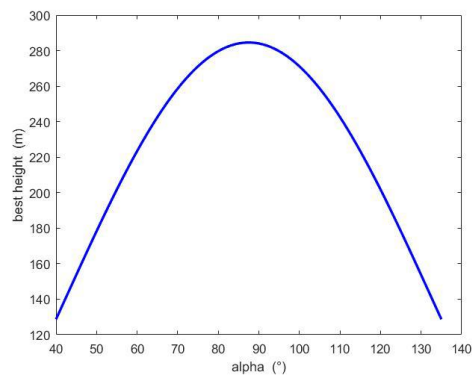

Figure 15 Simulation results(e)

4. The model establishment and solution of task2

4.1 Problem Analysis

When the drone is photographing the target object, the time it takes for the object to stay in the field of view is different because it is moving toward the target object and shooting at different angles.The camera has a certain frame rate, and if the object stays in the field of view for a short period of time, there may be ghosting and blurring.Therefore, for the fourth problem, we need to establish a mathematical model to solve the maximum time that the object can stay in the field of view of the camera at different angles to solve the best shooting angle.

Through the simultaneous cone surface equation and the plane equation, we can easily know that the intersecting plane is elliptical. Since the drone is in a uniform motion state, the elliptical field of view also moves at a uniform speed.When the object is stationary in place, the time that affects the object staying in the line of sight is determined by the long axis of the ellipse.

It is not difficult to find that the larger the inclination angle, the longer the long axis of the ellipse. Therefore, it is not difficult to find from the qualitative analysis that the time to stay when the inclination is maximum is necessarily the longest.However, when the inclination angle is too large, the area of   the ellipse is too large, that is, the field of view is too large. If the angle is too large, the target object cannot be seen. Therefore, we need to make a compromise between the dwell time and the field of view of the object. Get the best angle.

4.2 Model establishment and solution

4.2.1 Solving residence time

In the above, we have solved the UAV's field of view on the ground as an ellipse, considering that the drone only moves in the direction of the object, as shown below:
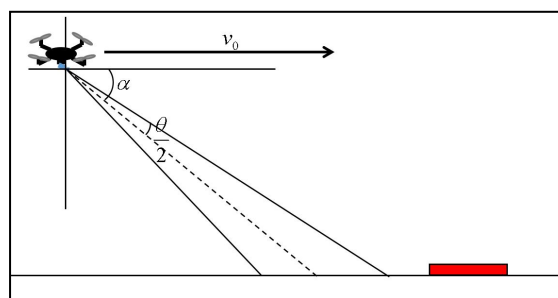

Figure 16 Geometrical analysis (a)

With the movement of the drone, the object is first seen at the forefront of the field of view.
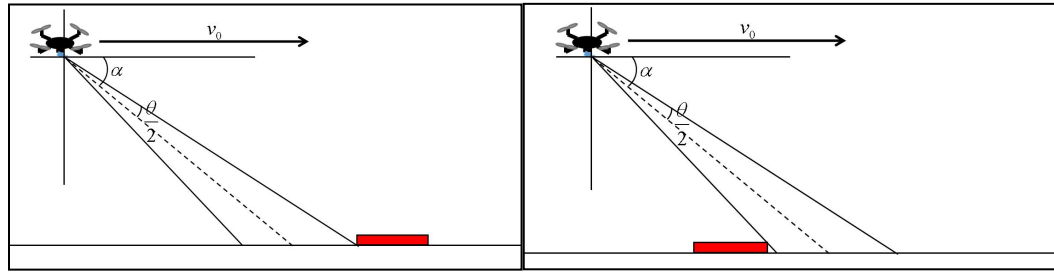
Figure 17 Geometrical analysis (b,c)

Remember this time $t = t_1$ As the drone continues to move forward until the object completely disappears into the sight of the drone, at this time $t = t_2$. From the ground direction, it is not difficult to find that the change of vision has gone through the process shown in the following figure:
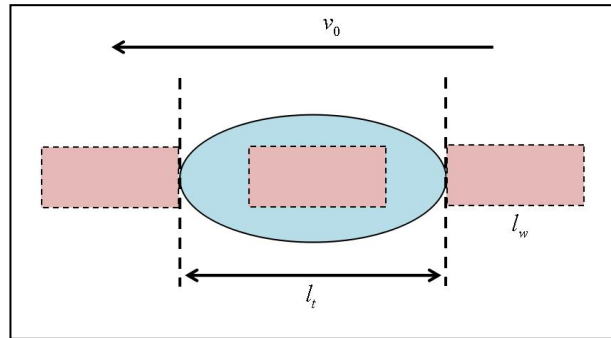
Figure 19 Geometrical analysis(d)

As can be seen from the figure, the residence time $\Delta t = t_2 - t_1$ The distance the object moves is the long semi-axis of the ellipse and its length $d = l_t + l_w$ From which you can conclude that the stay time:

$$t_d = \frac{d}{v_0} \tag{2.1}$$

Ellipse long axis l_t Can be converted to a standard form by elliptic equations $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Calculate and convert to standard form

$$l_t = 2a$$

4.2.2 Solving the maximum area of   view area

When defining the optimal shooting height, it is necessary to satisfy the shooting field of view to completely cover the object to be photographed. Therefore, when solving the maximum area ratio, it is only required to specify the area of   the elliptical field of view in the current situation.

The ellipse can be divided into upper and lower parts along the long axis. Solving the ellipse area can write the upper and lower parts as y on x The function is to integrate the upper and lower limits of the two parts of the function, and the two parts of the area are summed to obtain the total area of the ellipse.

The ellipse can be written as:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Write it as y on x The function

$$y^2 = \frac{a^2 b^2 - b^2 x^2}{a^2} \tag{2.2}$$

Then there are:

$$\begin{cases} y_1 = \sqrt{\dfrac{a^2 b^2 - b^2 x^2}{a^2}} \\ y_1 = -\sqrt{\dfrac{a^2 b^2 - b^2 x^2}{a^2}} \end{cases} \tag{2.3}$$

Calculate the integral, and set the midpoint of the ellipse to $C(x_c, y_c)$, to integrate the above formula, there are:

$$S_1 = \int_{x_c-a}^{x_c+a} \sqrt{\frac{a^2 b^2 - b^2 x^2}{a^2}}\, dx \tag{2.4}$$

The ellipse is a symmetrical figure, so there are:

$$S = 2S_1 \tag{2.5}$$

Ensure that when the line of sight is perpendicular to the ground, the object can still be completely covered by the elliptical area, so that at the angle, the total area of the object is smaller than the area of the elliptical field of view, and the area of the object is S_a Therefore, the maximum area ratio is $\lambda = \dfrac{S_a}{S}$ 。

4.3 Simulation experiment

In this simulation, the size of the object to be photographed is 20m*20m, and the camera with a focal length of 30mm is selected, and the angle of view is 30 degrees.The center of the square object is always on the camera line of sight, adjusting the camera height h To a fixed height h_s To explore the corner α Proportion of the largest area $\lambda = \dfrac{S_a}{S}$ With stay time $t_d = \dfrac{d}{v_0}$.

When solving long semi-axes, there are:

$$\frac{|\tan \alpha \cdot x - h|}{\sqrt{\tan^2 \alpha + 1}} = \sin \frac{\theta}{2} \sqrt{x^2 + y^2 + h^2} \tag{2.6}$$

In Equation 2.6, $y = 0$ Can get about x Equation

$$\frac{|\tan \alpha \cdot x - h|}{\sqrt{\tan^2 \alpha + 1}} = \sin \frac{\theta}{2} \sqrt{x^2 + h^2} \tag{2.7}$$

Solving Equation 2.7, you can get two solutions x_1, x_2

Through the calculation method above, the corner can be obtained α With stay

time $t_d = \dfrac{d}{v_0}$ The relationship is as shown below:
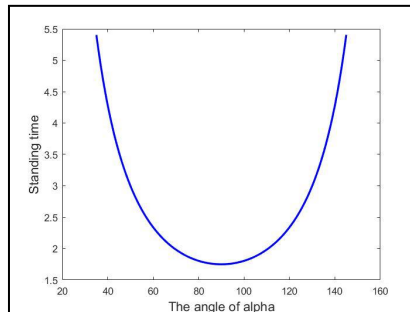
Figure 20 Geometrical analysis(a)

Next, the maximum area ratio is solved. First, the original elliptic equation is written as 1.41, and then the definite integral is solved. In the above, we find the abscissa of the two endpoints on the long axis of the ellipse. x_1, x_2, by substituting it into the upper and lower limits of the integral, the ellipse area can be obtained.

Through the calculation method above, the corner can be obtained α Proportion of the largest area $\lambda = \dfrac{S_a}{S}$ The relationship is as shown below:
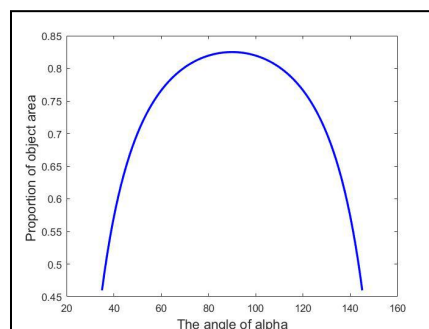

Figure 21 Geometrical analysis (b)

In order to better select the appropriate range of angles, we will combine the two images and get the results as shown below:
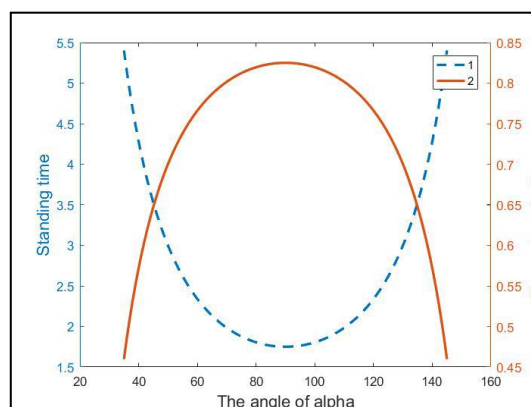
Figure 22 Geometrical analysis(c)

As can be seen from the above figure, the maximum area ratio and dwell time are negatively correlated. Considering many factors, in order to get the best shooting results, we define the optimal angle range between 35-55 and 125-145 degrees.At this point, the answer to question two ends.

5. The model establishment and solution of task3

5.1 problem analysis

Assuming that the drone is always moving in the direction of the object movement, there are three things that happen:

In the first case, the speed of the drone $v_f < v_o$ In this case, the vision of the drone will be farther and farther, until the object is too far away from the unmanned, the shooting loses its meaning.The second case, the speed of the drone $v_f = v_o$ In this case, the shooting angle will remain unchanged.The third case, the speed of the drone $v_f > v_o$ At this time, the vision of the drone will be from far to near, and then from near to far.

The second case loses the value of the research, so this article only solves the third case.During the entire flight, the drone will inevitably go through the process of chasing the target to surpassing the target. Therefore, in order to obtain the best shooting effect, the angle and shooting height must be continuously captured.By analyzing the geometric relationship, this paper establishes a differential equation model, which mainly explores that the object moves forward by the same distance at different times, and the UAV needs to adjust the angle and height.
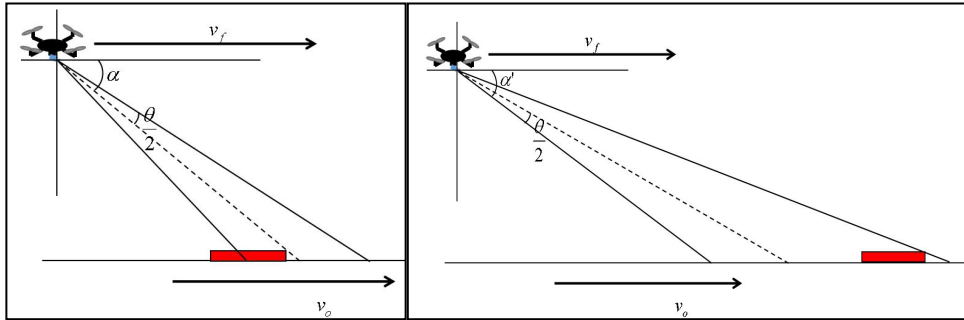
5.2 Model establishment and solution

Figure 23 Flight process analysis

In order to ensure that the object is always within the field of view, the drone must constantly adjust the angle to make it within the shooting range.In order to ensure the quality of the shot, it is not effective to set the shot if the area occupied by the subject in the field of view is less than 1%.

Since the area of the subject is not changed at all times, the shooting is meaningless when the field of view of the drone is 100 times the area of the subject.

From Equation 1.45, we can know that the field of view taken by the drone is about height. h Corner α Angle of view θ The function.Therefore, the area of the ellipse can be written as follows:

$$S_t = f(h, \alpha, \theta) \tag{4.1}$$

Obviously, for the third problem, the height of the drone flight h Perspective θ Are constant, and under critical conditions, $S_t = 100 S_O$, S_O For known object area, therefore, is known h , θ ,

S_O In the case, it is inevitable that the maximum angle can be solved by the equation α_{max} 。 Now

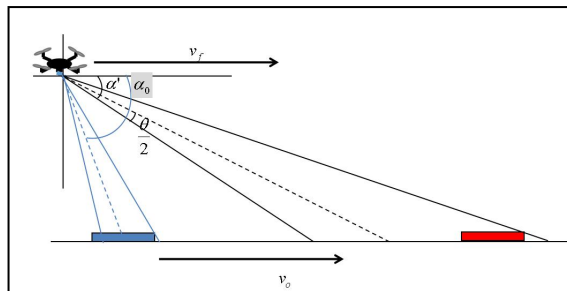need to be at the maximum corner α_{max} Solve the corner α The law of change.

Figure 24 Flight process analysis(b)

The speed of movement of an object relative to a drone is:

$$v' = v_o - v_f$$

After awhile Δt The object moves forward:

$$S = (v_o - v_f)v' \tag{4.2}$$

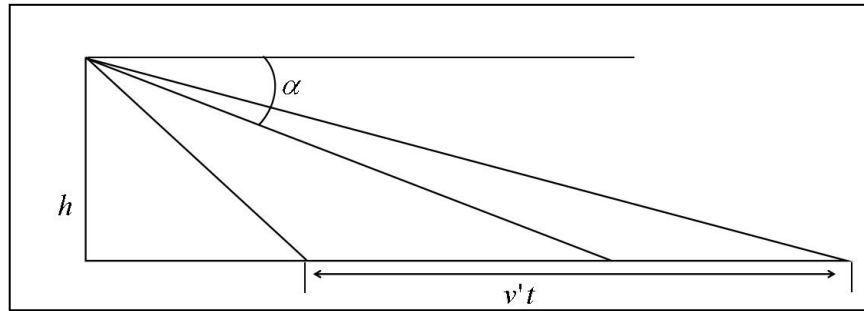Simplify the graph and get the results as shown below:

Figure 25 Flight process analysis(c)

At this point, after a very small time Δt, the object moves a small distance in the direction of velocity Δx, Filming angle α Have a particularly small reduction $\Delta \alpha$, has the following relationship:

$$\Delta x = \frac{h}{\tan(\alpha - \Delta\alpha - \frac{\theta}{2})} - \frac{h}{\tan(\alpha - \frac{\theta}{2})} \tag{4.3}$$

5.3 Simulation

In order to calculate the change law of the angle when the object moves, $\Delta x = 1$ And give h, θ Give appropriate values, explore α versus $\Delta \alpha$ Relationship, using matlab for programming operations, $\theta = 10°$, $h = 50$, get the following results.
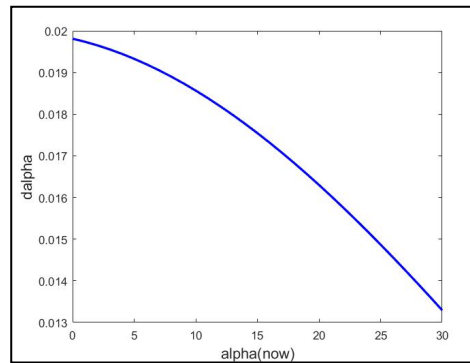
Figure 26 Simulation result

The above results show that when the line of sight starts to tilt from perpendicular to the ground, the angle of the field of view moving 1m requires the angle of the camera to be gradually reduced, which is also in line with common sense, the more the line of sight is inclined, the larger the field of view.

α For the current camera rotation angle, meet:

$$v't\tan\alpha = h \tag{4.4}$$

Therefore, there are:

$$\alpha = \arctan(\frac{h}{v't}) \tag{4.5}$$

Therefore, it can be concluded α Rate of change $\Delta \alpha$ And time t The relationship (object speed is 10m / s, drone speed is 5m / s) as shown below:

Figure 27 Simulation result

The figure above shows that the conversion rate of the angle per unit time decreases with the movement time of the object.So far, the solution to problem 3 ends.

6. The model establishment and solution of task4

6.1 Problem Analysis

Problem 4 assumes that there is an obstacle on the route of the drone. The drone needs to bypass the obstacle but keep the object shooting. Here, we build a obstacle model to carry out the entire movement of the drone. analysis.

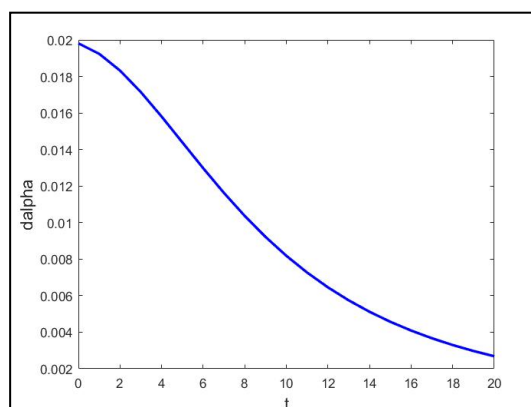Simulated environment: the size of the object to be photographed is 6m*6m, and the horizontal distance from the drone is 40m. On the road with a horizontal distance of 30m from the drone, there is an infinitely high cylindrical obstacle with a radius of 2m. The man-machine needs to bypass the cylindrical obstacle and keep shooting the object, and keep it at the best shooting height and angle.

The flight process of the drone can be divided into three sections:

The first step: the drone has not encountered obstacles, and maintains a uniform flight state to the target;The second step: the drone detects the obstacle, provides acceleration to the direction that needs to be detoured, and flies around the obstacle;The third step: the drone bypasses the obstacle and flies towards the target until it flies over the target.

Assume that the detection range of the drone is 4m. If the obstacle enters the detection range, the drone responds immediately, the side booster runs, and the speed direction is changed. During the whole process, the drone maintains the same speed and only changes the speed direction. .Through the solution process of Problem 1, we have learned that when in the best shooting situation, the height h With corner α The function relationship.Therefore, it is first necessary to solve the best flight speed by analyzing the process of bypassing the obstacle. v_b,

then analyze the shooting process, the best flight height h_b Relationship with flight time, and then

derive the best shooting angle α_b Relationship with flight time.

6.2 Model establishment, solution and simulation verification

The flight process of the drone is shown in the figure below.
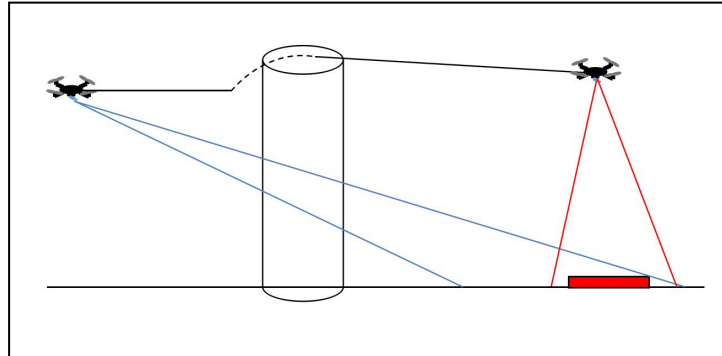
Figure 28 Flight path analysis

Set the initial height of the drone to 40m and substitute it into the result of question one to get the best initial corner angle of 43°.Inquire about the relevant information, and understand the principle of the straight-line driving and lateral driving of the drone as shown in the following figure:
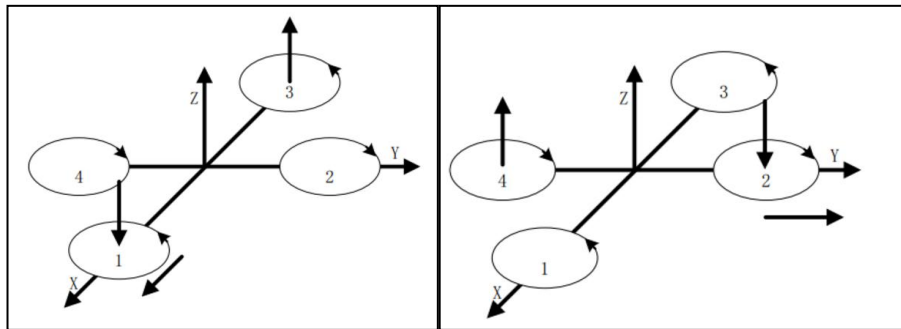
Figure 29 Uav flight principle

The steering motion of the drone can be equivalent to providing lateral thrust in a direction perpendicular to the direction of travel, providing instantaneous velocity in one side direction while providing an acceleration that is opposite to the speed in the side direction.Let the lateral acceleration that can be provided be $1 m/s^2$ At this time, the path at the time of steering can be equivalent to a parabola, and the following figure is a top view of the drone flying during the turn:
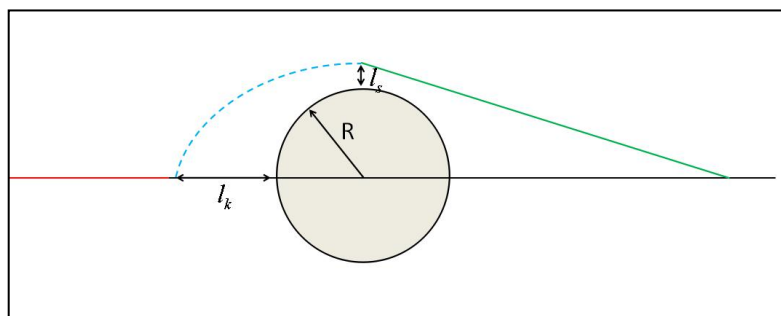
Figure 30 Process of circumventing obstacles

Decompose the velocity into horizontal and vertical directions:

$$vt = R + l_k$$
$$v_y t - 0.5at^2 = R + l_s \tag{5.1}$$

Among them l_k To detect the distance, l_s For the safety distance, here is 1m, and the simultaneous solution can be solved: $t = 6.45s, v = 1.3m/s$ .In the first phase, the drone is in a constant flight phase, using the model in question one:
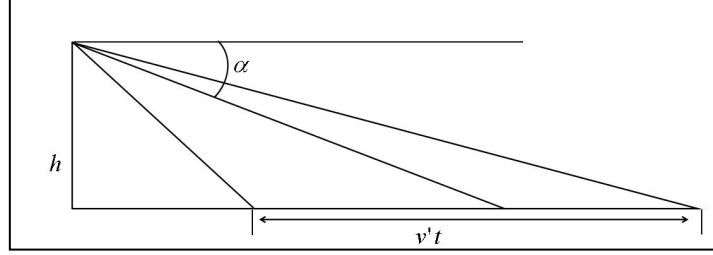
Figure 31

$$h = vt \tan(\alpha - \frac{\theta}{2}) \tag{5.2}$$

$$\alpha = \arctan(\frac{h}{vt}) + \frac{\theta}{2} \tag{5.3}$$

In a very short time, it can be regarded as height h Stay the same, iterate out a new one using Equation 5.2 α Value, then substituting into Equation 5.2, updating a new one h 。 The two-dimensional coordinates of the drone in the top view:

$$x(t) = vt$$
$$y(t) = v_y t - 0.5t^2$$

In the second phase, the UAV's line of sight is obscured by obstacles. At this time, there is also a velocity in the y direction. The coordinates of the target in the two-dimensional plane are $O(40,0)$,at this time α It should be calculated by the following formula:

$$\alpha = \arctan\left[\frac{(vt-40)^2 + (v_y t - 0.5t^2)}{h}\right] \tag{5.4}$$

As in the above method, it is updated in a very short time. h versus α Value.The third stage uses the first stage method, which is updated sequentially. h versus α Value.

In order to make the solution simple, directly apply the set of optimal angles and optimal heights that have been found in the first question. $h(1) = 128.6, a(1) = 40$ At this time, the distance from the object is 318m, and the final result is as shown in the following figure:
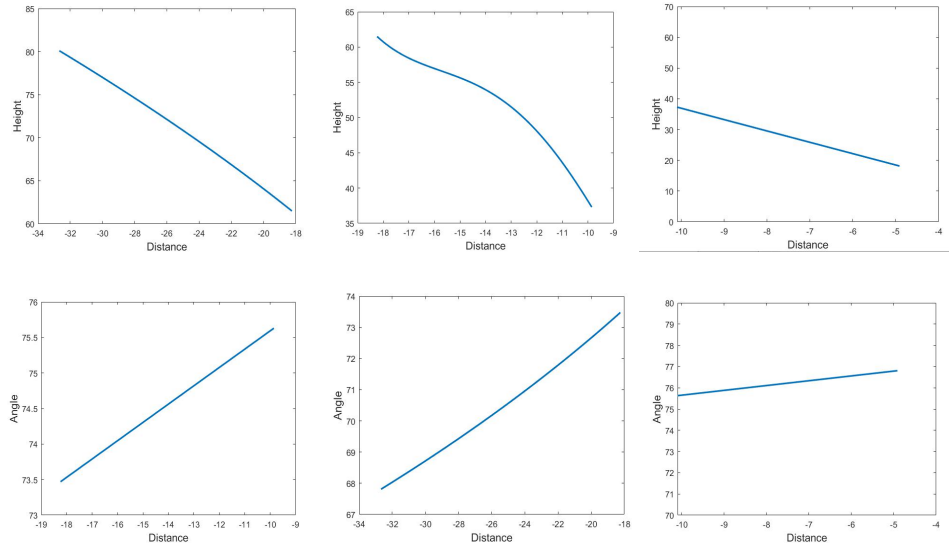
Figure 32Simulation result

Place the simulation results of the three phases in the same graph and get the following results:
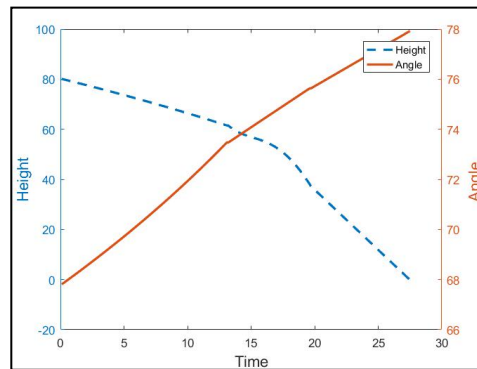
Figure 33 Simulation result

As can be seen from the above figure, the corner is in an enlarged state and is rotated perpendicular to the ground. In order to achieve a better shooting effect, the height is lowered.

7 .Model evaluation

Advantages: The models established in this paper are based on accurate geometric analysis, so the model has high credibility.Experiments using matlab simulations verify the accuracy of the established model.

Disadvantages: This paper lacks the analysis of the trajectory of dealing with complex obstacles, and the ability to deal with complex terrain is insufficient.

This model can be widely applied to all kinds of aircraft shooting work, and the improved model of the UAV steering can make the model more accurate and reliable.

References

[1] Trajectory design and control for aggressive formation flight with quadrotors[J] . Matthew Turpin,Nathan Michael,Vijay Kumar.   Autonomous Robots . 2012 (1)

[2] Precise quadrotor autonomous landing with SRUKF vision perception. Yang S,Ying J,Lu Y,et al. 2015 IEEE International Conference on Robotics and Automation(ICRA) . 2015

[3] Quadrotor Helicopter Trajectory Tracking Control. Hoffmann G M,Waslander S L,Tomlin C J. AIAA2008-7410 . 2008

[4] Visual Odometry and Mapping for Autonomous Flight Using an RGB-D Camera. Huang A S,Bachrach A,Henry P,et al. Robotics Research . 2011

Appendix

Code

```
clc
clear
f=10:80;
theta=180*2*atan(6./(f.*2));
plot(f,theta,'linewidth',2)
set(gcf,'color','white');
xlabel('focal length (mm)','fontsize',13);
ylabel('angle of view (℃)','fontsize',13);

clc
clear`
x2=(2*k*(y1-b)-x1*(k^2-1))/(k^2+1);
y2=(2*(b+x1*k)+y1*(k^2-1))/(k^2+1);
x=0:0.1:5;
y=k*x;
plot(x,y);
hold on
xx=[x1 x2];
yy=[y1 y2];
scatter(xx,yy);
axis equal
clc
clear
figure
[x,y,z] = meshgrid(-10:0.1:10,-20:0.1:0,-10:0.1:10);
v =(-((z-10)+y))/(2^0.5)-(x.^2+y.^2+(z-10).^2).^0.5*cos(pi*10*0.5/180);
p = patch(isosurface(x,y,z,v,0),'FaceColor','r','EdgeColor','none');
isonormals(x,y,z,v,p)
view(3)
axis equal
axis([-10 10 -20 0 -10 10])
set(gcf,'color','white')
grid on
hold on
xx=x(:,:,1);
yy=y(:,:,1);
zz=zeros(201,201);
Mesh(xx,yy,zz,'FaceColor','b','EdgeColor','none') % draw the surface z=0
```

```matlab
alpha(0.5)
xlabel('x');
ylabel('y');
zlabel('z');

clc
clear
figure
al=pi*120/180;
th=pi*20/180;
h=50;
[x,y,z] = meshgrid(0:200,-40:40,-10:100);
xx=x(:,:,1);
yy=y(:,:,1);
zz=zeros(size(xx));

for all=0:-1:-90
Mesh(xx,yy,zz,'FaceColor','b','EdgeColor','none') % draw the surface z=0
axis([0 200 -40 40 -100 100])
view(3)
set(gcf,'color','white')
grid on
alpha(0.5)
xlabel('x');
ylabel('y');
zlabel('z');
axis equal
al=all*pi/180
v
=((abs(tan(al).*x+(z-h)))/((tan(al)^2+1)^0.5))-((cos(th/2))*(x.^2+y.^2+(z-h).^2)
.^0.5);
p = patch(isosurface(x,y,z,v,0),'FaceColor','r','EdgeColor','none');
hold off
pause(0.1)
cla
end

 clc
clear
figure
al=pi*60/180;
th=pi*10/180;
h=50;
```

```matlab
[x,y,z] = meshgrid(-200:0,-40:40,-100:100);
   v
=((abs(tan(al).*x-h))/((tan(al)^2+1)^0.5))-((cos(th/2))*(x.^2+y.^2+h^2).^0.5);
p = patch(isosurface(x,y,z,v,0),'FaceColor','r','EdgeColor','none');
% isonormals(x,y,z,v,p)
view(3)
axis equal
set(gcf,'color','white')
grid on
hold on
axis([-150 -50 -40 40 -100 100])

alpha(0.5)
xlabel('x');
ylabel('y');
zlabel('z');
clc
clear
n=1;
warning off
figure
nnmin=40;
nnmax=135;
s=solve('(sin(th/2))^2*((h/tan(al)-25)^2+(25*1.25)^2+h^2)=(tan(al)*((h/ta
n(al)-25))-h)^2/(tan(al)^2+1)','h');
for ql=nnmin:nnmax

    al=ql*pi/180;
    th=10*pi/180;
    sr=eval(s);
    if sr(1)>0
            h(n)=sr(1);
    else
            h(n)=sr(2);
    end

    n=n+1;
end

all=nnmin:nnmax;
plot(all,h,'-b','linewidth',2)
set(gcf,'color','white')
xlabel('alpha    (°)');
ylabel('best height    (m)');
```

```
clc
clear
n=1;
warning off
figure
nnmin=45;
nnmax=135;

for ql=nnmin:nnmax

s=solve('(sin(th/2))^2*((h/tan(al)-25)^2+(25*1.25)^2+h^2)=(tan(al)*((h/tan(al)
-25))-h)^2/(tan(al)^2+1)','h');
        al=ql*pi/180;
        th=10*pi/180;
        sr=eval(s);
        if sr(1)>0
                    h(n)=sr(1);
        else
                    h(n)=sr(2);
        end

        n=n+1;
end

all=nnmin:nnmax;
plot(all,h,'-b','linewidth',2)
set(gcf,'color','white')


[x,y,z] = meshgrid(0:200,-40:40,-10:500);
xx=x(:,:,1);
yy=y(:,:,1);
zz=zeros(size(xx));

figure
set(gcf,'color','white');
for n=length(h):-1:1
        al=all(n)*pi/180;
        th=th;
        H=h(n);
        A=[-h(n)/tan(al)+15,15];
        B=[-h(n)/tan(al)+15,-15];
```

```matlab
        C=[-h(n)/tan(al)-15,-15];
        D=[-h(n)/tan(al)-15,15];
        fi44(A,B,C,D)
        xlabel('X');
        ylabel('Y');
        zlabel('Z');
        axis([0 200 -40 40 -10 10])
        axis equal
        al=(90-all(n))*pi/180;
        v
=((abs(tan(al).*x+(z-H)))/((tan(al)^2+1)^0.5))-((cos(th/2))*(x.^2+y.^2+(z-H).^
2).^0.5);
        p = patch(isosurface(x,y,z,v,0),'FaceColor','r','EdgeColor','none');
            alpha(0.5)
        pause(1.5)
    end

    clc
    clear
    th=10*pi/180;
    all=35:145;
    s=solve('(sin(th/2))^2*(x^2+h^2)=(tan(al)*x-h)^2/(tan(al)^2+1)','x');
    for i=1:length(all);
        al=all(i)*pi/180;
        h=50;
        x=eval(s);
        lt(i)=abs(x(1)-x(2));
        td(i)=lt(i)/5;
    end
    plot(all,td,'-b','linewidth',2)
    xlabel('The angle of alpha','fontsize',13);
    ylabel('Standing time','fontsize',13);
    set(gcf,'color','white');

    sr=-td;
    sr=sr.*0.1+1;
    figure
    plot(all,sr,'-b','linewidth',2);
    set(gcf,'color','white');
    xlabel('The angle of alpha','fontsize',13);
    ylabel('Proportion of object area ','fontsize',13);


    figure
```

```
[AX,H1,H2]=plotyy(all,td,all,sr);
set(gcf,'color','white');
set(get(AX(1),'Ylabel'),'string','Standing time','fontsize',13);
set(get(AX(2),'Ylabel'),'string','Proportion of object area ','fontsize',13);
xlabel('The angle of alpha','fontsize',13);
set(H1,'linestyle','--','linewidth',2)
set(H2,'linestyle','-','linewidth',2)
legend('1','2');
```

```
clc
clear
all=90:-0.1:45;
h=50;
x(1)=0;
for i=1:length(all);
    al(i)=all(i)*pi/180;
    x(i+1)=x(i)+h/(tan(al(i)-pi/1000-10*pi/180))-h/(tan(al(i)-10*pi/180));
end
al=pi/2-al;
al=al*180/pi;
plot(x(2:length(x)),al)
```

```
clc
clear
da=solve('x=h/(tan(a-da-th/2))-h/(tan(a-th/2))','da');
th=10*pi/180;
aa=60:90;
h=50;
x=1;
for i=1:length(aa)
    a=aa(i)*pi/180;
    dd(i)=eval(da)
end
plot(90-aa,dd,'b-','linewidth',2)
set(gcf,'color','white');
xlabel('alpha(now)','fontsize',13);
ylabel('dalpha','fontsize',13);
```

```
clc
clear
da=solve('x=h/(tan(arctan(h/(v*t))-da-th/2))-h/(tan(arctan(h/(v*t))-th/2))','da');
th=10*pi/180;
tt=0:20;
h=50;
x=1;
v=5;
for i=1:length(tt)
        t=tt(i);
        dd(i)=eval(da)
end
plot(tt,dd,'b-','linewidth',2)
set(gcf,'color','white');
xlabel('t','fontsize',13);
ylabel('dalpha','fontsize',13);


clc
clear
h(1)=222.7;
a(1)=40*pi/180;
x(1)=318;
v1=1.3;
t1=x(1)/v1;
dt=0.1;
th=10*pi/180;
tt=0.01:dt:t1;

for i=1:length(tt)
        a(i+1)=atan(h(i)/(318-v1*tt(i)))+th/2;
        h(i+1)=tan(a(i)-th/2)*(318-v1*tt(i));
        x(i+1)=h(i+1)/tan(a(i+1));
end

aa=a(2131:2261);
ha=aa.*180/pi;
hh=h(2131:2261);
```

```
xx=-x(2131:2261);
td=0:130;
plot(xx,hh,'linewidth',2)
set(gcf,'color','white')
xlabel('Distance','fontsize',13);
ylabel('Height','fontsize',13);
figure
plot(xx,ha,'linewidth',2)
set(gcf,'color','white')
xlabel('Distance','fontsize',13);
ylabel('Angle','fontsize',13);


clc
clear
h(1)=61.5;
x(1)=18.24;
y(1)=0;
dt=0.1;
t=0:dt:6.45;
vy=4;
a(1)=73.47*pi/180;
th=10*pi/180;
for i=2:length(t)
    y(i)=y(i-1)+vy*dt;
    vy=vy-1*dt;
    x(i)=x(i-1)-1.3*dt;
    a(i)=atan(h(i-1)/sqrt((y(i-1))^2+(x(i-1))^2))+th/2;
    h(i)=tan(a(i)-th/2)*sqrt((y(i))^2+(x(i))^2);
    d(i-1)=sqrt(x(i)^2+y(i)^2+h(i)^2);
end
aa=linspace(a(1),1.32,length(t));
plot(-x,h,'linewidth',2)
set(gcf,'color','white')
xlabel('Distance','fontsize',13);
ylabel('Height','fontsize',13);

figure
plot(-x,aa.*180/pi,'linewidth',2)
set(gcf,'color','white')
xlabel('Distance','fontsize',13);
ylabel('Angle','fontsize',13);
```

```
clc
clear
x(1)=9.85;
y(1)=5.03;
d(1)=sqrt(x(1)^2+y(1));
h(1)=37.30;
v=1.3;
tm=d(1)/1.3;
dt=0.1;
t=0:dt:tm;
for i=1:length(t)
    d(i+1)=d(i)-dt*v;
    a(i+1)=atan(h(i)/d(i));
    h(i+1)=tan(a(i+1))*d(i+1);
end
plot(-d,h,'linewidth',2)
axis([-10.1 -4 0 70])
set(gcf,'color','white')
xlabel('Distance','fontsize',13);
ylabel('Height','fontsize',13);

figure
aa=linspace(1.32,1.36,length(d));
plot(-d,aa.*180/pi,'linewidth',2)
axis([-10.1 -4 70 80])
set(gcf,'color','white')
xlabel('Distance','fontsize',13);
ylabel('Angle','fontsize',13);
```

Experimental Data

h1=[80.1063403302710,79.9805782382275,79.8528824423031,79.726715036
0650,79.5986175020063,79.4720409088641,79.3435377506010,79.21654803
55513,79.0876353041877,78.9602284685199,78.8309021509042,78.7030741
307463,78.5733301479983,78.4450768128202,78.3149110188148,78.186228
1698862,78.0556363496919,77.9265197184953,77.7954975867658,77.66594
28333602,77.5344860326769,77.4044887440125,77.2725928431779,77.1421
485313598,77.0098090236364,76.8789131241341,76.7461254254308,76.614
7732952328,76.4815327422339,76.3497196579446,76.2160215061804,76.08
37426620572,75.9495820839145,75.8168325898423,75.6822046725107,75.5
489795519131,75.4138792952661,75.2801734829488,75.1445957973568,75.

0104041372813,74.8743438413533,74.7396610843383,74.6031129025906,74.4679337039383,74.3308922643855,74.1952111814289,74.0576710130956,73.9214825026655,73.7834380330146,73.6467364488203,73.5081820010953,73.3709615910173,73.2318913814942,73.0941462847854,72.9545544199306,72.8162786643209,72.6761591378515,72.5373466365529,72.3966933263947,72.2573378750019,72.1161445401417,71.9762398134240,71.8345000906503,71.6940396392302,71.5517470397591,71.4107242866721,71.2678721926517,71.1262804297837,70.9828620906723,70.8406944750668,70.6967030038801,70.5539525539111,70.4093809233316,70.2660405147358,70.1208815530781,69.9769439148398,69.8311903018657,69.6866480119477,69.5402922745244,69.3951377554380,69.2481722630308,69.1023977772375,68.9548147372318,68.8084123823675,68.6602038352092,68.5131655391254,68.3643233532728,68.2166408688825,68.0671567355603,67.9188216354824,67.7686870632283,67.6196907342183,67.4688970432120,67.3192306803697,67.1677689965332,67.0174235972766,66.8652848461349,66.7142512039284,66.5614261042187,66.4096948020423,66.2561738590588,66.1037352626063,65.9495087612670,65.7963530118592,65.6414110094805,65.4875280166775,65.3318603354415,65.1772397693412,65.0208359884387,64.8654672716404,64.7083167190743,64.5521890182945,64.3942807623237,64.2373829796413,64.0787058198462,63.9210265835600,63.7615690415091,63.6030966965852,63.4428470060803,63.2835696041652,63.1225157010431,62.9624209900206,62.8005505014851,62.6396259145502,62.4769261480070,62.3151587922292,62.1516167235971,61.9889933679432,61.8245956294105,61.6611026921963,61.4958355593886];

a1=[67.8098805506040,67.8464530731908,67.8836124491628,67.9203513221311,67.9576768924271,67.9945836851755,68.0320770237181,68.0691533300706,68.1068160353973,68.1440634741461,68.1818971699583,68.2193173854220,68.2573237211503,68.2949183837485,68.3330990351345,68.3708698419795,68.4092265116745,68.4471751871803,68.4857096053622,68.5238379018717,68.5625518268802,68.6008615253112,68.6397567443018,68.6782496548127,68.7173279844313,68.7560059471068,68.7952692341840,68.8341341197418,68.8735842420093,68.9126379525289,68.9522768193577,68.9915212890315,69.0313508421931,69.0707880381012,69.1108102525530,69.1504421754624,69.1906590601576,69.2304877453465,69.2709013440702,69.3109288621794,69.3515412544111,69.3917697123215,69.4325830141258,69.4730145558659,69.5140309208123,69.5546677284943,69.5958893486065,69.6367336433934,69.6781627501315,69.7192167932365,69.7608556585098,69.8021217522294,69.8439726894450,69.8854531782259,69.9275185433720,69.9692158149155,70.0114980076816,70.0534144940845,70.0959159590203,70.1380541379560,70.1807773656705,70.2231397616109,70.2660872900130,70.3086764754931,70.3518508910759,70.3946694880035,70.4380734271744,70.4811241081852,70.5247602586438,70.5680457485066,70.6119168506712,70.6554399277430,70.6995487762300,70.7433122739646,70.7876617191215,70.8316685276334,70.8762614771289,70.9205145448146,70.9653539652876,71.0098563005088,71.0549452192792,71.0996998921087,71.1450413989534,71.

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    h2=[61.5000000000000,61.0765701745161,60.6818444157325,60.3141810102058,59.9719166922821,59.6533732906196,59.3568642400808,59.0807008850843,58.8231985097041,58.5826820398489,58.3574913733516,58.1459863043486,57.9465510186102,57.7575981461912,57.5775723667087,57.4049535705416,57.2382595861987,57.0760484899628,56.9169205186963,56.7595196104253,56.6025346000675,56.4447000995376,56.2847970925408,56.1216532747719,55.9541431700835,55.7811880525759,55.6017557036073,55.4148600315089,55.2195605804084,55.0149619530906,54.8002131713199,54.5745069955680,54.3370792246911,54.0872079948039,53.8242130954531,53.5474553202223,53.2563358681182,52.9502958115340,52.6288156462610,52.2914149389549,51.9376520876733,51.5671242116110,51.1794671869934,50.7743558472755,50.3515043673737,49.9106668536653,49.4516381639882,48.9742549849062,48.4783971971649,47.9639895646133,47.4310037870202,46.8794609632852,46.3094345186502,45.7210536578236,45.1145074155861,44.4900493876320,43.8480032372991,43.1887690886256,42.5128309329924,41.8207651955702,41.1132506288706,40.3910797237271,39.6551718525333,38.9065883846212,38.1465500376952];

    a2=[1.28229340144023,1.28288256704273,1.28347173264523,1.28406089824772,1.28465006385022,1.28523922945272,1.28582839505521,1.28641756065771,1.28700672626020,1.28759589186270,1.28818505746520,1.28877422306769,1.28936338867019,1.28995255427269,1.29054171987518,1.29113088547768,1.29172005108018,1.29230921668267,1.29289838228517,1.29348754788766,1.29407671349016,1.29466587909266,1.29525504469515,1.29584421029765,1.29643337590015,1.29702254150264,1.29761170710514,1.29820087270764,1.29879003831013,1.29937920391263,1.29996836951512,1.30055753511762,1.30114670072012,1.30173586632261,1.30232503192511,1.30291419752761,1.30350336313010,1.30409252873260,1.30468169433510,1.30527085993759,1.30586002554009,1.30644919114258,1.30703835674508,1.30762752234758,1.30821668795007,1.30880585355257,1.30939501915507,1.30998418475756,1.31057335036006,1.31116251596255,1.31175168156505,1.31234084716755,1.31293001277004,1.31351917837254,1.31410834397504,1.31469750957753,1.31528667518003,1.31587584078253,1.31646500638502,1.

```
31705417198752,1.31764333759001,1.31823250319251,1.31882166879501,1.
31941083439750,1.32000000000000];
    a2=a2.*180/pi;
    h3=[37.3000000000000,36.8200009696816,36.3400019393632,35.86000
29090448,35.3800038787264,34.9000048484081,34.4200058180897,33.9400
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0106664978,31.5400116361794,31.0600126058610,30.5800135755426,30.10
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2600223026772,25.7800232723588,25.3000242420404,24.8200252117220,24.
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4200300601301,21.9400310298117,21.4600319994933,20.9800329691750,20.
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46];
    a3=[1.32000000000000,1.32051282051282,1.32102564102564,1.321538
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4102564103,1.32615384615385,1.32666666666667,1.32717948717949,1.327
69230769231,1.32820512820513,1.32871794871795,1.32923076923077,1.32
974358974359,1.33025641025641,1.33076923076923,1.33128205128205,1.3
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35846153846154,1.35897435897436,1.35948717948718,1.36000000000000];
```

```matlab
a3=a3.*180/pi;
h=[h1 h2 h3];
a=[a1 a2 a3];
t=0.1:0.1:27.5;
[AX,H1,H2]=plotyy(t,h,t,a);
set(gcf,'color','white');
set(get(AX(1),'Ylabel'),'string','Height','fontsize',13);
set(get(AX(2),'Ylabel'),'string','Angle ','fontsize',13);
xlabel('Time','fontsize',13);
set(H1,'linestyle','--','linewidth',2)
set(H2,'linestyle','-','linewidth',2)
legend('Height','Angle');
```