

Team Control Number

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Problem Chosen

**B**

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Summary Sheet

## Summary

UAV is often used for high altitude shooting, widely used in military, civil and science and technology fields, and has become a research hotspot at home and abroad. In order to solve the problems of altitude, angle and tracking state of UAV, we establish Photographic Height Determination Model, Shooting Angle Model, Drone Tracking Model and other models.

**In task 1**, according to the characteristics of UAV squint shooting, we analyze the relationship between flying altitude and angle and image resolution, and establish the UAV aerial photography altitude model. The range of UAV's height is higher than the range of the optimal aerial height. The results show that the range of the best shooting height varies from 110 to 333m according to the angle. When the shooting angle is fixed, the shooting height is higher than the best shooting height at this time.

**In task 2**, the actual ground conditions discussed in this paper are two types: high-rise building and mountain area. By setting the optimal aerial shooting height as the objective function, the photographing area of UAV is studied, the shooting angle model of multi-objective planning is established, and the optimal range of UAV shooting angle is obtained. The results show that: the smaller the UAV shooting angle is to ensure the maximum richness of information collection. The range of the best aerial angle should be less than the optimum angle given in the table 2.

**In task 3**, the drone that tracks the specific motion trajectory of the ground has three linear motions (front and rear, up and down, left and right) and three angular motions (rolling, pitching, and yaw). According to its dynamics and kinematics, the drone can be vertical and horizontal small disturbance state space equation. According to this, the angle of photography of the drone in each equation of state is corrected.

**In task 4**, the obstacle envelope is constructed to obstruct the space enveloping surface, and by finding the corresponding interface, a series of path points meeting the requirements are solved, and the feasible paths are fitted to the interpolation of these points. In the UAV simulation experiment, the obstacle avoidance trajectory is obtained by using the two methods of overriding the terrain obstacle and bypassing the terrain obstacle reasonably.

**Key word:** Oblique photography, Multiobjective planning, Trajectory tracking, Obstacle Avoidance

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# 1. Introduction

## 1.1 Background

With the development of science and technology, more and more UAVs are used in people's daily work and study. UAV is often used for high altitude shooting, widely used in military, civil and science and technology fields, and has become a research hotspot at home and abroad.

UAV aerial photography has obvious advantages in the process of practice. With the help of advanced positioning technology and technology support, it can truly reflect and analyze the situation of the tested object, thus building a three-dimensional real-world model to study the characteristics and properties of the tested object. When the UAV is shooting, it needs a certain height from the ground. The higher the flight altitude is, the larger the range of camera acquisition will be, but the pixel points per unit area will be less, and the shooting accuracy will be lower. In addition, the speed and angle of the flight also have an impact on the shooting effect. How to use UAV to take pictures with large amount of information and high precision is one of the problems to be solved.

## 1.2 Work

The UAV's flying altitude, flying speed and shooting angle affect its shooting range, and then affect the accuracy of the collected information. In order to study the influence of flying height, flying speed and shooting angle on UAV aerial photography, make UAV take satisfactory photos and collect as rich and accurate data as possible, we need to use mathematical knowledge to solve the modeling of UAV aerial photography.

The problems that we need to solve in this paper are:

- Assumed that the UAV flies in a straight line and is not affected by wind direction, humidity and other external factors during the flight process, and the flying speed and shooting angle remain unchanged. We calculate the altitude range of UAV and propose the best aerial photographing height at the specified shooting angle.
- Assumed that the UAV flight altitude and speed are fixed and the UAV needs to perform more accurate image acquisition on the actual ground conditions. We will calculate the optimal range of UAV shooting angle.
- Assumed that the UAV flight altitude and speed are fixed, and the UAV is collecting the track of a moving object on the ground. We will calculate the change rule of UAV shooting angle.
- When the UAV encounters obstacles in the flight process, we should calculate the optimal flying speed, flying altitude and shooting angle, and provide relevant data simulation experiments.

## **2. Problem analysis**

### **2.1 Analysis of question one**

In view of the first problem, the topic requires the UAV to study the flight altitude under the condition of constant flight speed and shooting angle. It is found that UAV squint photography is widely used in aerial photography because of its high accuracy. Therefore, this paper studies the aerial photographing behavior of UAV in the case of tilt photographing. Because the accuracy of camera acquisition information is reflected by resolution, the best aerial photographing height of the whole area that can be photographed by UAV can be obtained by establishing the model of resolution, flight height and angle in the research, and the height range is the range higher than this optimal aerial photographing height. And the optimal height of the aerial photograph under different flight angles is obtained.

### **2.2 Analysis of question two**

In view of the second problem, it is required to study the flight angle of UAV when the flight speed and shooting height of UAV are constant. Problem 1 has calculated the optimal height of UAV in different aerial angles. In the actual ground conditions, the optimal height of UAV changes with the actual terrain fluctuation. The actual ground conditions discussed in this paper are high-rise building area and mountain area. By setting the optimal aerial height as the objective function, the shooting area of UAV is studied, and the optimal range of UAV shooting angle is obtained.

### **2.3 Analysis of question three**

For the task three, the mission requires the UAV to fly at a fixed speed and altitude, and is tracking the object on the ground for a specific motion trajectory to calculate the change in the shooting mode of the drone. The drone has three linear motions (front and rear, up and down, left and right) and three angular motions (rolling, pitching, yaw). According to its dynamics and kinematics feature, Small disturbance state space equation, which involves with the longitudinal and lateral directions of the final moving object of the drone, can be obtained.

### **2.4 Analysis of question four**

In the task four, the mission requires the drone to avoid obstacles on the path, calculate the optimal flight speed, flight altitude and photographic angle, and conduct data simulation experiments. We use obstacles (signal towers, utility poles, and houses) as peaks, fit the terrain parabola function, construct the obstacle space envelope surface, and find a series of path points that meet the requirements by finding the corresponding interface. These point interpolations fit the feasible path.

The curvature formula is used to calculate the direction angle and the rate of change of the climb angle, which can be applied to the UAV dynamics and kinematics model for further track inspection.

### 3. Symbol and Assumptions

#### 3.1 Symbol Description

Symbols	Definition
$H$	The photographing height
$f$	The focal length of the tilt camera
$d$	Sensor cell size
$\gamma$	Each pixel angle
$\alpha$	Specified shooting angle
$\beta$	Half of field angle
$\Delta H_o$	The best difference between UAV height and building height
$\theta$	Shooting angle
$L$	The building spacing
$\Delta V$	The increment of the speed vector
$\Delta \theta$	The roll angle of the drone
$\Delta \delta_z$	the small disturbance

#### 3.2 Fundamental assumptions

To simplify the given problems and modify it more appropriate for simulating real-life conditions, we make the following basic hypotheses, each of which is properly justified.

- We assume that the drone can be regarded as a rigid body with six degrees of freedom, ignoring the elastic deformation of the drone, and its motion can be seen as including three linear motions (front and rear, up and down, left and

right) and three angular motions (rolling, pitch, yaw).

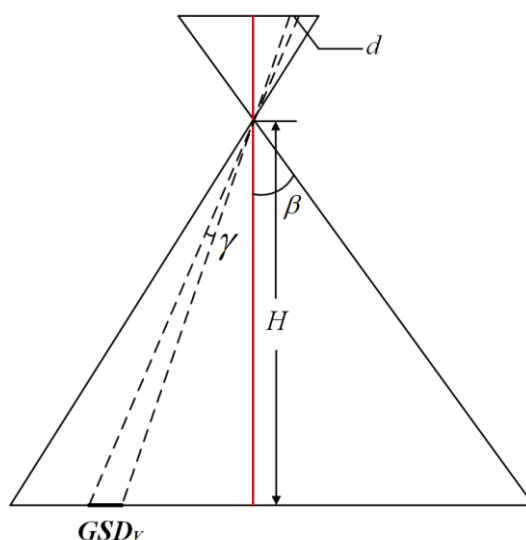
- We assume that the gravitational acceleration does not change with the change in flight altitude, and that the Earth is considered an inertial system, ignoring the curvature of the Earth and treating the earth as a plane.

## 4. Model

### 4.1 Task 1: Photographic Height Determination Model

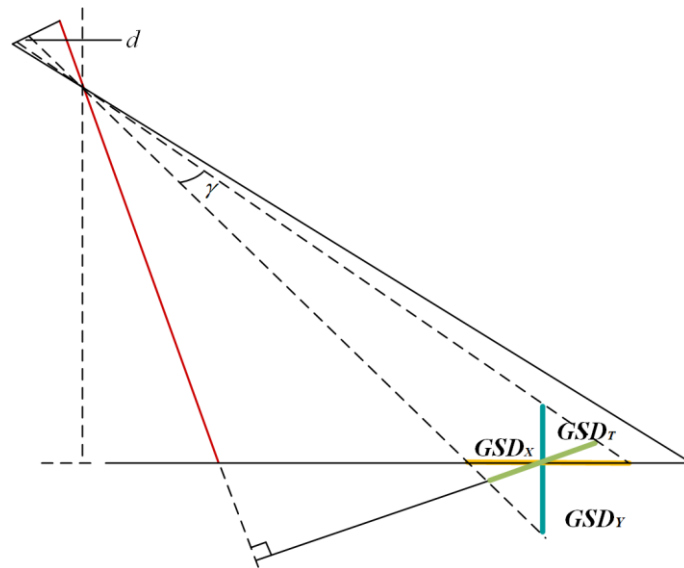
#### 4.1.1 UAV Height Determination

Tilt photography includes two angles of view: front view and squint. The resolution of front view is the resolution of the photo taken when the front view camera is perpendicular to the ground. The definition of frontal resolution is shown in Figure 2. The red line represents the main optical axis.



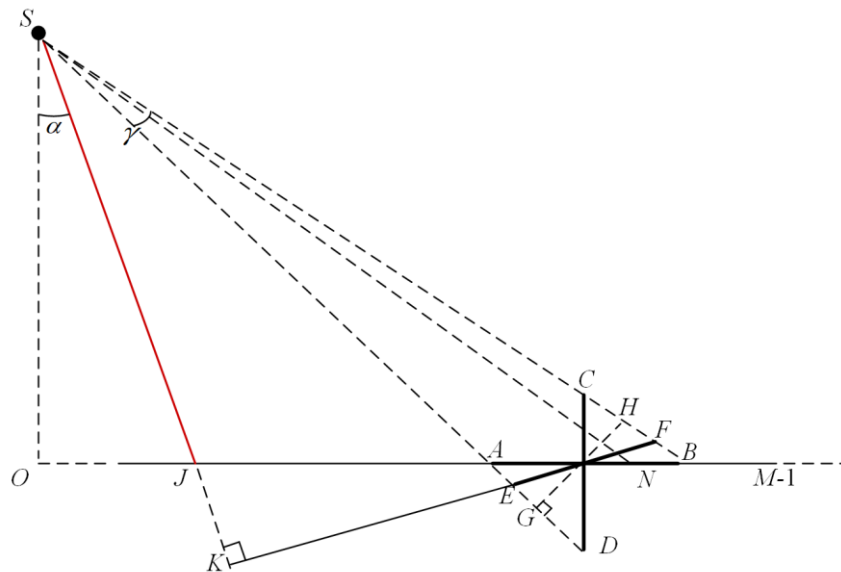
**Figure.2** Front photography.

Squint can not only get the side information of the object, but also the top information without occlusion. It is widely used in aerial photography. There are differences between vertical and horizontal resolutions. When squint photography is performed<sup>[1]</sup>, the squint resolution is the size of the image element of the tilted camera on a plane perpendicular to the main optical axis. Assumed that the tilt camera projects to the horizontal plane, according to the principle of central projection, the ground coverage of the resulting photograph gradually increases from near to far, so the resolution increases from small to large (Fig. 3).



**Figure.3** Squint photography. Yellow line, green line and blue line represent horizontal resolution, oblique scale and vertical resolution respectively.

Assume that the sensor cell size is  $d$ , the focal length of the tilt camera is  $f$ , the photographing height is  $H$ , the specified shooting angle is  $\alpha$ , the field of view angle is  $2\beta$ , each pixel angle is  $\gamma$ ,  $EF$  is the  $m_{th}$  ( $0 \leq m \leq M-1$ ,  $M$  is the number of the pixels) pixel from on the intersection line. As shown in Figure 4, red line in the figure represents the main optical axis.



**Figure.4** Strabismus photography resolution diagram.

According to the central projection principle, if the center point of the photograph is  $S$ , then the main optical axis of the camera is  $\alpha$  ( $\angle SOK = \alpha$ ) inclined. The plumb line is coplanar with the main optical axis, and the angle between the  $m_{th}$  pixel  $EF$  on the image plane and the vertical direction is

$$\angle OSN = \alpha - \beta + m\gamma \tag{4.1}$$

Through geometric relationship projected by central projection principle, the

length of the pixel is

$$EF = \frac{d \times SK}{f} \quad (4.2)$$

And combining the angle  $(\alpha - \beta + m\gamma)$  between the  $m_{ih}$  pixel and the vertical direction and the angle  $(\beta - m\gamma)$  between the first pixel and the main optical axis, the distance of the light source  $S$  along the main optical axis to the pixel  $EF$  is

$$SK = \frac{H \cos(\beta - m\gamma)}{\cos(\alpha - \beta + m\gamma)} \quad (4.3)$$

According to the geometric relationship  $\angle ENG = \angle FNH = \beta - m\gamma$ , it will be converted to the vertical direction, there is

$$GH = EF \cos(\beta - m\gamma) \quad (4.4)$$

The projections of  $GH$  in the horizontal and vertical directions are

$$\begin{cases} AB = \frac{GH}{\cos(\alpha - \beta + m\gamma)} \\ CD = \frac{GH}{\sin(\alpha - \beta + m\gamma)} \end{cases} \quad (4.5)$$

For low altitude UAV aerial camera system, the relationship between image ground resolution and aerial height is: when the aerial height is higher, the image ground resolution is lower; otherwise, when the aerial height is lower, the image ground resolution is relatively higher. When the focal length is  $f$  that the camera of the drone is photographing the ground at a high altitude  $H$  from the ground, the front view resolution  $GSD_v$  is

$$GSD_v = \frac{d \times H}{f} \quad (4.6)$$

When the camera of the drone has a specific tilt angle  $\alpha$  with the vertical direction, it is known from equations (4.2) and (4.3) that the squint resolution  $GSD_r$  is

$$GSD_r = \frac{d \times SK}{F} = \frac{d \times H \cos(\beta - m\gamma)}{f \cos(\alpha - \beta + m\gamma)} \quad (4.7)$$

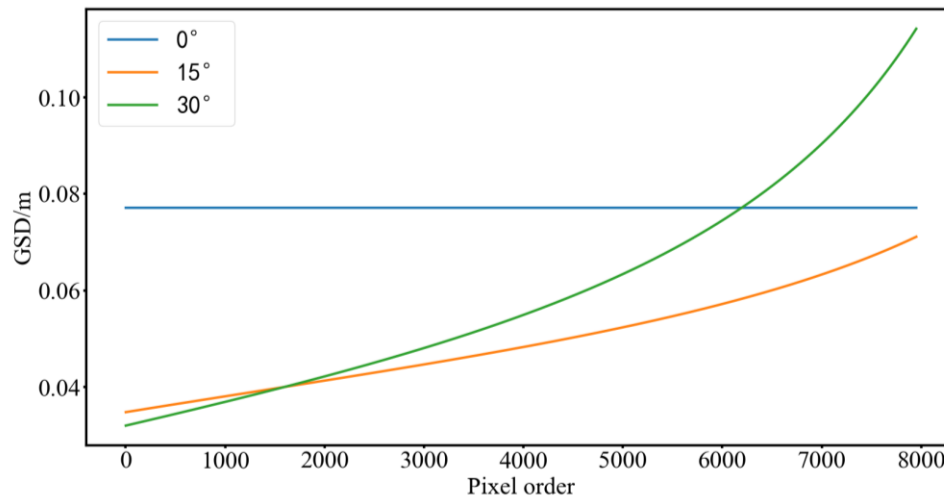
The corresponding squint resolutions are respectively decomposed into horizontal and vertical directions to obtain squint horizontal resolution  $GSD_x$  and squint vertical resolution  $GSD_y$  as



$$\begin{cases} GSD_x = \frac{d \times H \cos(\beta - m\gamma) \cos(\beta - m\gamma)}{f \cos(\alpha - \beta + m\gamma) \cos(\alpha - \beta + m\gamma)} \\ GSD_y = \frac{d \times H \cos(\beta - m\gamma) \cos(\beta - m\gamma)}{f \cos(\alpha - \beta + m\gamma) \sin(\alpha - \beta + m\gamma)} \end{cases} \quad (4.8)$$

#### 4.1.2 UAV Altitude Range and Optimum Height at Fixed Angle

When the angle of UAV aerial photography changes, the resolution of pixels at different positions will also change. In the process of researching the accuracy of UAV aerial photography at different angles, it is of certain significance to average the resolution of different pixels for the research of UAV aerial photography accuracy.



**Figure.5** GSD of different pixel points at 0 °, 15 °, 30 °.

Figure 5 shows that when the camera tilt angle of UAV is 0 ° no tilt occurs, the resolution of each pixel point is the same, and its resolution is around 0.08m. When the tilt angle of the camera increases gradually, the resolution difference of the pixel points increases gradually, which shows that the farther the distance between the pixel points is, the smaller the resolution is.

Assumed that the range of UAV shooting is 200m, the optimal aerial height of UAV is calculated according to different aerial angles. When the flight height of UAV in each angle is greater than the optimal height, the UAV can complete shooting of the designated area.

**Table1: Optimum shooting height at different angles.**

Slope ( ° )	0	5	10	15
Optimum height (m)	195.62	332.80	325.23	312.88
Slope ( ° )	20	25	30	35
Optimum height (m)	296.12	275.45	251.51	225.02
Slope ( ° )	40	45	50	55
Optimum height (m)	196.79	167.67	138.56	110.33

In fact, when the tilt angle of UAV camera exceeds  $55^\circ$ , the shooting result will not be complete, so the angle range is set within  $55^\circ$ . Table 1 shows the optimal shooting height of the UAV when the angle changes from  $0^\circ$  to  $55^\circ$  and the range of this optimal shooting height varies from 110 to 333m according to the angle. When the shooting angle is fixed, the shooting height is higher than the best shooting height at this time, that is, when the shooting angle is fixed at  $0^\circ$ , the range of the optimal shooting height is greater than 110.33m.

## 4.2 Task 2: Shooting Angle Under Elevation Transformation

### 4.2.1 Aerial Angle of High-rise Building Area

When the UAV is working in the high-rise building area, on the premise of avoiding the collision between the UAV and the building, in order to get as many shooting areas as possible and the shooting accuracy as high as possible, and at the same time to ensure that the highest building does not block other buildings. Combined with the optimal aerial height of UAV in task one, taking the optimal difference between UAV height and building height as the objective function, the aerial angle model of high-rise building area is established:

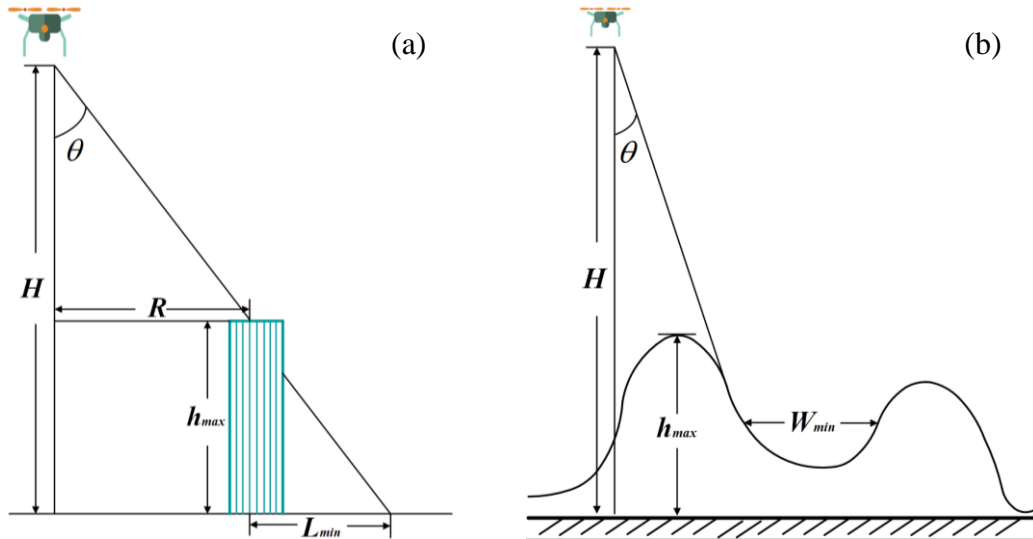
Objective Function

$$\min |\Delta H - \Delta H_o| \quad (4.9)$$

Restraint Condition

$$\begin{cases} \Delta H = H - h \\ \tan \theta = \frac{R}{\Delta H} \\ H \tan \theta - R \leq L_{\min} \\ h \leq h_{\max} \end{cases} \quad (4.10)$$

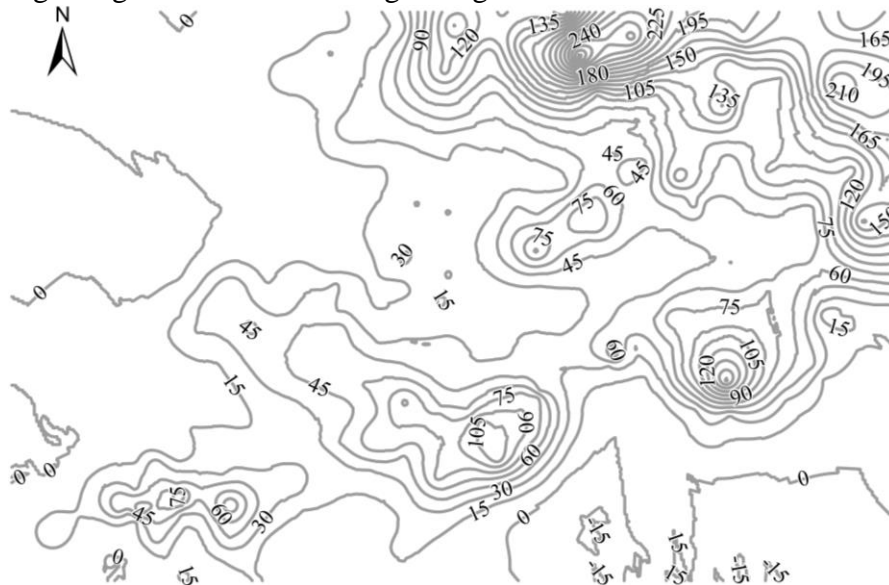
In the formula,  $H$  is the flight height of UAV,  $h$  is the height of high-rise building,  $\Delta H_o$  is the best difference between UAV height and building height,  $\Delta H$  is the difference between UAV height and building height,  $\theta$  is the UAV shooting angle,  $L$  is the building spacing,  $R$  is the maximum horizontal distance between UAV [2] and building, that is to say, UAV can achieve aerial photography in the circular area with  $R$  as the radius, without covering Block other buildings.



**Figure.6** Shooting process under actual ground conditions. Figure (a) shows the shooting process when the shooting environment is a high-rise building, and figure (b) shows the shooting process in the mountains.

#### 4.2.2 Aerial Angle of Mountain

Compared with the high-rise building area, the mountainous area also has the characteristics of great elevation change. The difference between the two is that the terrain of the mountainous area fluctuates continuously. In this paper, the local longitude, latitude and altitude information of a city as shown in Figure 7 are selected, and the altitude change of the city is observed to be about 250m, so as to study the optimal flight angle when the UAV flight height is fixed.



**Figure.7** Contour map of study area.

When the operation area is mountainous area or hilly area, the most ideal mode is to fly according to the specific relative height according to the height fluctuation of the ground because of its complex topography<sup>[3]</sup>. However, the assumption of this problem is that the UAV's flight height is fixed. In this regard, the uplifted hills are equivalent to high-rise buildings, and the sunken valleys are equivalent to open spaces

between high-rise buildings. At this time, the valley width is regarded as the space between high-rise buildings, and the aerial photographing process is shown in Figure 6b.

Similar to the aerial photographing angle model of high-rise building area, the aerial photographing angle model of mountainous area is

Objective Function

$$\min |\Delta H - \Delta H_o| \quad (4.11)$$

Restraint Condition

$$\begin{cases} \Delta H = H - h \\ \tan \theta = \frac{R}{\Delta H} \\ H \tan \theta - R \leq W_{\min} \\ h \leq h_{\max} \end{cases} \quad (4.12)$$

In the formula,  $H$  is the flight height of UAV,  $h$  is the height of mountains,  $\Delta H_o$  is the best difference between UAV height and mountains,  $\Delta H$  is the difference between UAV height and mountains height,  $\theta$  is the UAV shooting angle,  $W$  is the valley width,  $R$  is the maximum horizontal distance between UAV and mountains.

### 4.2.3 Result display

The aerial photographing angle of UAV in the complex terrain environment affects the richness of information collected by UAV. According to the actual experience, it can be concluded that the larger the terrain fluctuation changes, the higher the requirements for the aerial photographing angle of UAV, and the calculated results also reflect such results.

**Table2: Optimum shooting angle at a certain altitude height**

Height ( $m$ )	100	120	140	160
Optimum angle ( ° )	62.88	60.04	57.36	54.78
Height ( $m$ )	180	200	220	240
Optimum angle ( ° )	52.29	49.85	47.45	45.07
Height ( $m$ )	260	280	300	320
Optimum angle ( ° )	42.68	40.28	37.85	35.36

From table 2, it can be seen that the larger the relief is, the smaller the UAV shooting angle is to ensure the maximum richness of information collection. The range of the best aerial angle should be less than the optimum angle given in the table.

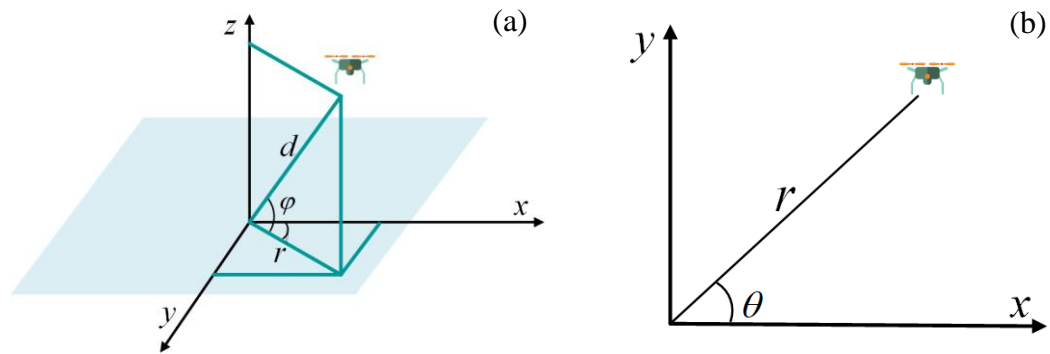
### 4.3 Task 3: Drone Tracking Model

The drone is the most important controlled object in the process of tracking the ground moving target object. Its motion can be seen as including three line motions (front and rear, up and down, left and right) and three angular motions (rolling,

pitching, Yaw), its dynamics and kinematics can be described by 12 first-order nonlinear differential equations<sup>[4]</sup>. The theoretical basis of the drone model is the state space description of the system, which describes the equation as follows:

$$\dot{X} = g(t, X, u) \quad (4.13)$$

The state quantities  $X = [\beta, \eta, \phi, \beta_w, \eta_w, \phi_w, v_U, v_V, v_W, X_e, Y_e, H]^T$  respectively indicate a roll angle, a pitch angle, a yaw angle, a roll angular velocity, a pitch angular velocity, a yaw angular velocity, a velocity component on the body coordinate axis, a longitudinal displacement, a lateral displacement, and a height;  $u = [\delta_a, \delta_b]^T$ , present aileron declination, rudder angle;  $t$  present time.



**Figure.8** Schematic diagram of drone tracking ground targets(a)Three-dimensional stereo view (b) 2D plan view.

The differential equations are classified according to the longitudinal and lateral directions, simplifying the order of the flight control system, and the longitudinal constant coefficient differential equation and the lateral constant coefficient differential equation formula can be obtained. The expression of the space equation for the side of the small disturbance of the drone is as follows:

$$\dot{X} = AX + BU \quad (4.14)$$

For longitudinal movement:

$$\begin{cases} X^v = [\Delta V \ \Delta \alpha \ \Delta \theta \ \Delta \omega_z \ \Delta H]^T \\ U^v = [\Delta \delta_z \ \Delta \delta_p] \end{cases} \quad (4.15)$$

Among them,  $\Delta V$  is the increment of the speed vector,  $\Delta \alpha$  is the drone angle of the drone,  $\Delta \theta$  is the roll angle of the drone,  $\Delta \omega_z$  is the projection of the attitude to the axis  $Z$  of the drone,  $\Delta H$  is the increment of the aeronautical altitude,  $\Delta \delta_z$  and  $\Delta \delta_p$  are the small disturbance. For the longitudinal movement, the specific parameters of  $A^v$  and  $B^v$  are as follows:

$$A^v = \begin{bmatrix} -n_{1V} & -n_{1\alpha} & -n_{1\theta} & 0 & 0 \\ -n_{21} & -n_{2\alpha} & -n_{2\theta} & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ n_{2V}n_{3\alpha} - n_{3V} & n_{2\alpha}n_{3\alpha} - n_{3\alpha} & n_{2\theta}n_{3\alpha} & -n_{3\theta} - n_{3\alpha} & 0 \\ n_{4V} & -n_{4\alpha} & n_{4\theta} & 0 & 0 \end{bmatrix}, B^v = \begin{bmatrix} 0 & 0 \\ -n_{2\delta_2} & 0 \\ 0 & 0 \\ n_{2\delta_1}n_{3\alpha} - n_{3\delta_1} & 0 \\ 0 & 0 \end{bmatrix} \quad (4.16)$$

For lateral movement:

$$\begin{cases} X^h = [\beta \ \omega_x \ \omega_y \ \gamma]^T \\ U^h = [\delta_x \ \delta_y]^T \end{cases} \quad (4.17)$$

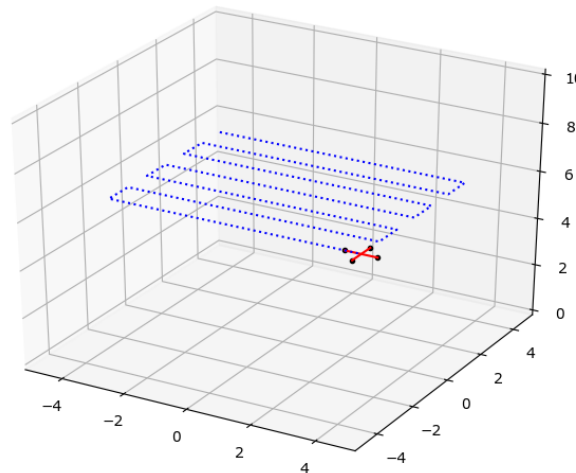
Among them,  $\beta$  is the side slip angle,  $\omega_x$  and  $\omega_y$ , which are the projection of the three-axis attitude of the drone to the axis  $X$  and axis  $Y$  of the aircraft,  $\gamma$  is the yaw angle. The specific parameters  $A^h$  and  $B^h$  for the lateral motion and are as follows:

$$A^h = \begin{bmatrix} -n_{1\beta} & -n_{1\omega_y} & 1 & -n_{1\phi} \\ -n_{2\beta} & -n_{2\omega_x} & -n_{2\omega_1} & 0 \\ -n_{3\beta} & -n_{3\omega_x} & -n_{3\omega_y} & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, B^h = \begin{bmatrix} 0 & -n_{1\delta_y} \\ -n_{2\delta_x} & -n_{2\delta_y} \\ 0 & -n_{3\delta_y} \\ 0 & 0 \end{bmatrix} \quad (4.18)$$

Then the equation of state for the longitudinal and lateral movement of the drone is as shown in equation (4.15):

$$\begin{cases} \dot{X}^v = A^v X^v + B^v U^v \\ \dot{X}^h = A^h X^h + B^h U^h \end{cases} \quad (4.19)$$

The flight path and shooting angle of the UAV are consistent with the trajectory of the ground object, and the UAV tracking path shown in Figure 9 is obtained.



**Figure.9** UAV tracking route to the ground.

#### 4.4 Obstacle Avoidance Trajectory Planning

Signal towers, utility poles, and houses are considered as mountain peaks. Using known topographic map data of a certain city, parabola fitting of these points will result in the parabolic function of the mountain topography as follows<sup>[5]</sup>:

$$\bar{Z} = \varphi(\bar{x}) = a\bar{x}^2 + b\bar{x} + c \quad (4.20)$$

Assume that the equation between the current node of the drone and the next node be  $k_{SG}x + b_{SG}$ , then get the simultaneous equations:

$$k_{SG}x + b_{SG} = ax^2 + bx + c \quad (4.21)$$

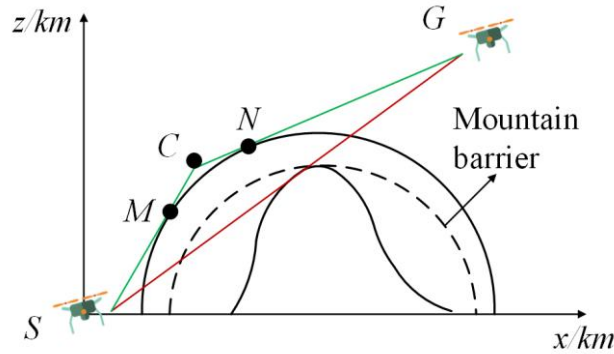
When the equation has a solution, it indicates that the SG needs to perform obstacle avoidance, that is  $(b-k)^2 - 4a(c-b_{SG}) \geq 0$ , when the flight chess track needs obstacle avoidance planning.

We propose a method to use space-based knowledge to find out the obstacles to avoid obstacles, namely to construct the obstacle envelope surface, and find a series of path points that meet the requirements by finding the corresponding interface, and fit the points to the interpolation. Feasible path. The curvature formula is used to calculate the direction angle and the rate of change of the climb angle, which can be applied to the UAV dynamics and kinematics model for further track inspection.

- Over the terrain obstacles

Assume that the straight lines SC and CG be  $k_{SC}x + b_{SC}$  and  $k_{CG}x + b_{CG}$ , then connect them separately to the parabola:

$$\begin{cases} k_{SC}(\bar{x} - \bar{x}_S) + z_S = a\bar{x}^2 + b\bar{x} + c + Z_{safe} \\ k_{CG}(\bar{x} - \bar{x}_G) + z_G = a\bar{x}^2 + b\bar{x} + c + Z_{safe} \end{cases} \quad (4.22)$$



**Figure.10** Over the terrain obstacles.

Among them,  $Z_{safe}$  is the safe distance, and the two equations have only one solution, according to the root formula, we have

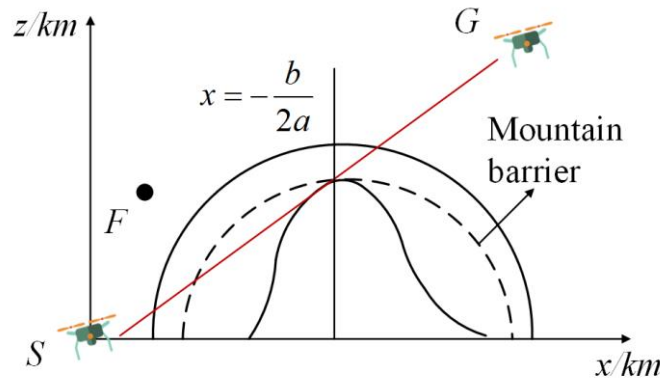
$$\begin{cases} (b - k_{SC})^2 - 4a(c + Z_{safe} - z_S + k_{SC}\bar{x}_S) = 0 \\ (b - k_{CG})^2 - 4a(c + Z_{safe} - z_G + k_{CG}\bar{x}_G) = 0 \end{cases} \quad (4.23)$$

Solve the intersection of two lines and a parabola. The trajectory composed of the line segments SM, NG and the parabolic segment MN is the trajectory of the terrain.

- Bypassing terrain barriers

Rotating the fitted quadratic parabola around its axis of symmetry yields a paraboloid, namely:

$$z = a \left[ \left( x - \frac{b}{2a} \cos \varphi_{SG} \right)^2 + \left( y - \frac{b}{2a} \sin \varphi_{SG} \right)^2 \right] + \frac{4ac - b^2}{4a} + Z_{safe} \quad (4.24)$$

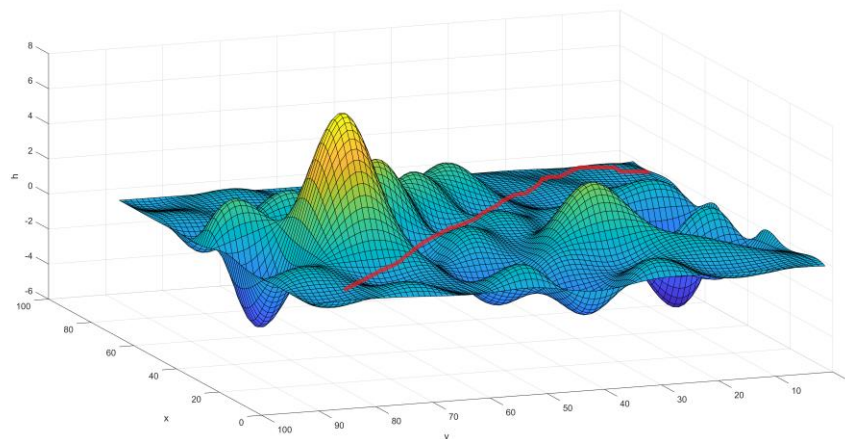


**Figure.11** Bypassing terrain barriers.

The plane equation can be obtained by passing the straight line SG for the plane of the plane OXZ:

$$l(x - x_s) + m(y - y_s) + n(z - z_s) = 0 \quad (4.25)$$

Then the space curve intersecting the parabola and the plane is the safe flight path of the aircraft.



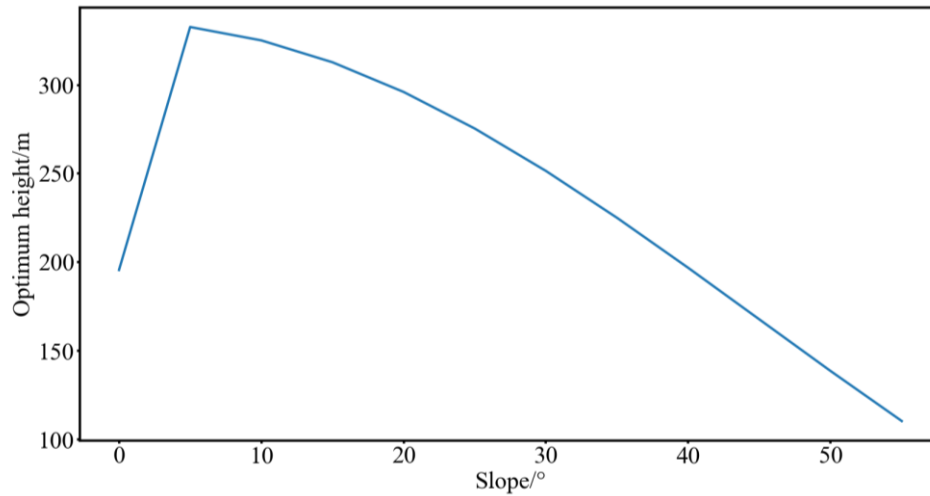
**Figure.12** Drone trajectory.

## 5. Test the Models

Assumed that the range of UAV shooting is 200m, the optimal aerial height of



UAV is calculated according to different aerial angles. When the flight height of UAV in each angle is greater than the optimal height, the UAV can complete shooting of the designated area.



**Figure.13** Optimum shooting height at different angles.

In the case of angle change, the test results shown in Figure 13 are consistent with the results shown in Table 1, and the model has better practicability.

## 6. Sensitivity Analysis

Through the above analysis, the richness of the information collected by the known UAV and the accuracy of the image taken show obvious negative correlation characteristics. The sensitivity of the model can be verified by adjusting the UAV shooting angle slightly, and the results are shown in Table 3.

**Table3: Change range of image accuracy at 0 °, 15 °, 30 °.**

Shooting angle(° )	0	15	30
Amount of change	2.78	1.27	2.47
Rate of change	4.59	9.43	6.01

When the shooting angle changes slightly, the change rate of imaging accuracy is less than 10%, and the model is relatively stable.

## 7. Strengths and Weakness

### 7.1 Strengths

- In this paper, the relationship between the richness of information collected by UAV and the accuracy of image is considered, and the model is more accurate, which is in line with the actual application.

- For the actual ground, the model established in this paper also has a certain guiding significance for the photography of different altitude mountain areas and different height building areas.

## 7.2 Weaknesses

- This paper does not consider the working environment and flight state of UAV, so there will be some errors in actual operation.
- When considering the characteristics of mountainous terrain, in order to simplify the model and simplify it to buildings, terrain elevation model can be used to make up for the defects in later research.

## 8. Conclusion

In this paper, from the information acquisition and accuracy of UAV aerial photography, through the control of UAV flight height and angle and other conditions, as well as the operation of UAV in different terrain conditions are studied. The model can provide guarantee for UAV to work in different environments under certain conditions, and can play a certain practical significance in practical tasks.

## References

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- [4] Wang Xun. Method and Experimental Research on autonomous tracking of ground moving target by UAV. *University of national defense science and technology*, 2012.
- [5] Jia Yangyang. Obstacle avoidance and path planning of aircraft. *Beijing University of technology*, 2015.

## Appendix

```

1. import numpy as np
2. import math
3. import matplotlib.pyplot as plt
4. import pandas as pd
5.
6. plt.rcParams['font.sans-serif'] = ['SimHei'] # 用来正常显示中文标签
7. plt.rcParams['axes.unicode_minus'] = False # 用来正常显示负号
8.
9. l, w = 7952, 5304
10. L = 200
11. d = 4.5E-6 #像元大小
12. fv, ft = 35E-3, 60E-3 #焦距
13. beltav, beltat = 54.4, 33.4 #视角
14. gamav, gamat = beltav / ( l / 2 ), beltat / ( l / 2 ) #像元视
    角
15. alpha = np.arange( 0, 50 ) #相机倾角
16. m = np.arange( 1, 1 )
17.
18.
19.
20. diff = np.zeros( 4 )
21. H = 600
22. a = np.array( [ math.radians( beltav - m[i] * gamav ) for i in range( len( m
    ) ) ] )
23. b = np.array( [ math.radians( alpha[0] - beltav + m[i] * gamav ) for i in ra
    nge( len( m ) ) ] )
24. GSD_X = np.array( [ ( ( d * H ) * ( math.cos( a[i] ) ** 2 ) ) / ( fv * ( mat
    h.cos( b[i] ) ** 2 ) ) for i in range( len( m ) ) ] )
25. diff[0] = np.max( GSD_X ) - np.min( GSD_X )
26. plt.figure()
27. plt.plot( m, GSD_X, label = '0°', linewidth = 3 )
28.
29. a = np.array( [ math.radians( beltat - m[i] * gamat ) for i in range( len( m
    ) ) ] )
30. b = np.array( [ math.radians( alpha[6] - beltat + m[i] * gamat ) for i in ra
    nge( len( m ) ) ] )
31. GSD_X = np.array( [ ( ( d * H ) * ( math.cos( a[i] ) ** 2 ) ) / ( ft * ( mat
    h.cos( b[i] ) ** 2 ) ) for i in range( len( m ) ) ] )
32. diff[1] = np.max( GSD_X ) - np.min( GSD_X )
33. plt.plot( m, GSD_X, label = '15°', linewidth = 3 )
34.

```

```
35. a = np.array( [ math.radians( beltat - m[i] * gamat ) for i in range( len( m
    ) ) ] )
36. b = np.array( [ math.radians( alpha[11] - beltat + m[i] * gamat ) for i in r
    ange( len( m ) ) ] )
37. GSD_X = np.array( [ ( ( d * H ) * ( math.cos( a[i] ) ** 2 ) ) / ( ft * ( mat
    h.cos( b[i] ) ** 2 ) ) for i in range( len( m ) ) ] )
38. diff[2] = np.max( GSD_X ) - np.min( GSD_X )
39. plt.plot( m, GSD_X, label = '30°', linewidth = 3 )
40.
41. a = np.array( [ math.radians( beltat - m[i] * gamat ) for i in range( len( m
    ) ) ] )
42. b = np.array( [ math.radians( alpha[16] - beltat + m[i] * gamat ) for i in r
    ange( len( m ) ) ] )
43. GSD_X = np.array( [ ( ( d * H ) * ( math.cos( a[i] ) ** 2 ) ) / ( ft * ( mat
    h.cos( b[i] ) ** 2 ) ) for i in range( len( m ) ) ] )
44. diff[3] = np.max( GSD_X ) - np.min( GSD_X )
45.
46. diff = diff * 1000
47. print( diff )
48. print( ( diff[ 1:4 ] - diff[ 0:3 ] ) / diff[ 0:3 ] )
49.
50. plt.legend( fontsize = 30 )
51. plt.xlabel( 'Pixel order', fontsize = 30, fontname = 'Times New Roman' )
52. plt.ylabel( 'GSD/m', fontsize = 30, fontname = 'Times New Roman' )
53. plt.xticks( fontsize = 30, fontname = 'Times New Roman' )
54. plt.yticks( fontsize = 30, fontname = 'Times New Roman' )
55. ax = plt.gca()
56. ax.spines['bottom'].set_linewidth( 3 )
57. ax.spines['left'].set_linewidth( 3 )
58. ax.spines['top'].set_linewidth( 3 )
59. ax.spines['right'].set_linewidth( 3 )
60. ax.tick_params( width = 3 )
61. # plt.savefig( '1.png', dpi=300, bbox_inches = 'tight' )
62. plt.show()
63. GSD = L / l
64. H = np.zeros( len( alpha ) )
65. a = np.array( [ math.radians( beltav - m[i] * gamav ) for i in range( len( m
    ) ) ] )
66. b = np.array( [ math.radians( alpha[0] - beltav + m[i] * gamav ) for i in ra
    nge( len( m ) ) ] )
67. a, b = np.mean( a ), np.mean( b )
68. H[0] = ( GSD * ( fv * ( math.cos( b ) ** 2 ) ) ) / ( d * ( math.cos( a ) **
    2 ) )
69. for i in range( 1, len(alpha) ):
```

```
70.     a = np.array( [ math.radians( beltat - m[j] * gamat ) for j in range( len( m ) ) ] )
71.     b = np.array( [ math.radians( alpha[i] - beltat + m[j] * gamat ) for j in range( len( m ) ) ] )
72.     a, b = np.mean( a ), np.mean( b )
73.     H[i] = ( GSD * ( ft * ( math.cos( b ) ** 2 ) ) ) / ( d * ( math.cos( a ) ** 2 ) )
74. plt.plot( alpha, H, linewidth = 3 )
75. plt.xlabel( 'Slope/°', fontsize = 30, fontname = 'Times New Roman' )
76. plt.ylabel( 'Optimum height/m', fontsize = 30, fontname = 'Times New Roman' )
77. plt.xticks( fontsize = 30, fontname = 'Times New Roman' )
78. plt.yticks( fontsize = 30, fontname = 'Times New Roman' )
79. ax = plt.gca()
80. ax.spines['bottom'].set_linewidth( 3 )
81. ax.spines['left'].set_linewidth( 3 )
82. ax.spines['top'].set_linewidth( 3 )
83. ax.spines['right'].set_linewidth( 3 )
84. ax.tick_params( width = 3 )
85. plt.show()
86. GSD = L / l
87. H = np.arange( 100, 400, 10 )
88. alphanrange = np.zeros( len( H ) )
89. for i in range( len( H ) ):
90.     degree = math.radians( beltat )
91.     alphanrange[i] = math.acos( math.sqrt( ( d * H[i] * ( math.cos( degree ) ** 2 ) ) / ( ft * GSD ) ) ) * ( 180 / math.pi ) + beltat - np.mean( gamat * m )
92. data = np.concatenate( [ H, alphanrange ], axis = 1 )
93. save = pd.DataFrame( list( data ) )
94. save.to_csv( 't2.csv' )
95. plt.plot( H, alphanrange, linewidth = 3 )
96. plt.xlabel( 'Slope/°', fontsize = 30, fontname = 'Times New Roman' )
97. plt.ylabel( 'Optimum height/m', fontsize = 30, fontname = 'Times New Roman' )
98. plt.xticks( fontsize = 30, fontname = 'Times New Roman' )
99. plt.yticks( fontsize = 30, fontname = 'Times New Roman' )
100. ax = plt.gca()
101. ax.spines['bottom'].set_linewidth( 3 )
102. ax.spines['left'].set_linewidth( 3 )
103. ax.spines['top'].set_linewidth( 3 )
104. ax.spines['right'].set_linewidth( 3 )
105. ax.tick_params( width = 3 )
106. plt.show()
```